



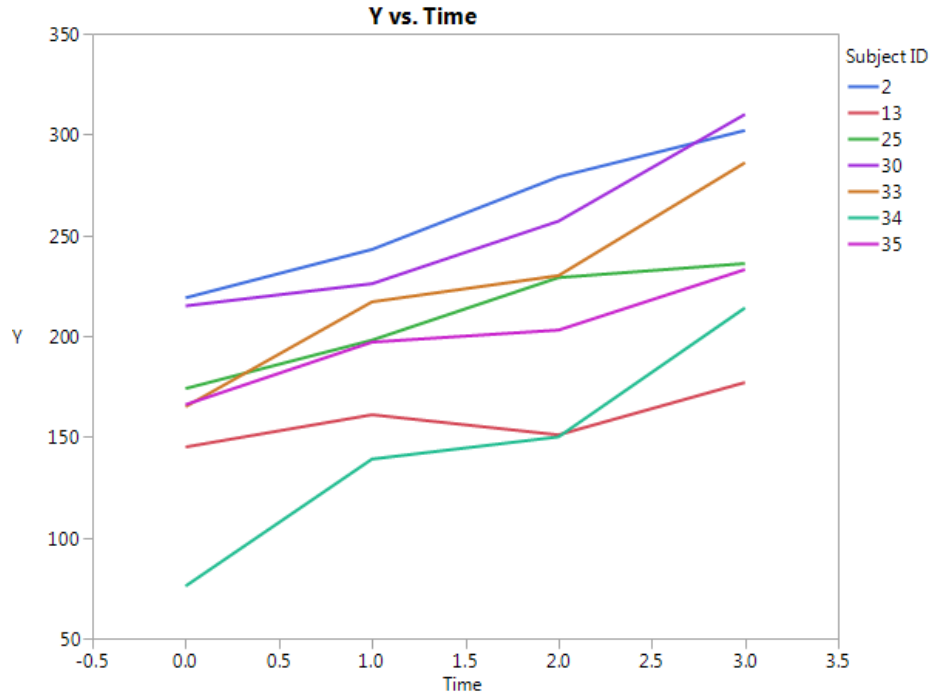
RANDOM COEFFICIENT MODELS

HOW TO MODEL LONGITUDINAL AND HIERARCHICAL DATA IN JMP® PRO



RANDOM COEFFICIENT MODELS

WHAT DO WE MEAN?



- Intercepts and slopes randomly distributed – potentially correlated
 - Hence “random coefficient”
- Multilevel situations
 - Observe students within schools
 - Commonly referred to as Hierarchical Linear Models (HLM)
- Hierarchical Bayes models
 - Similar in the Bayesian context
 - Not our focus today

WHY THE DIFFERENT TERMINOLOGY?

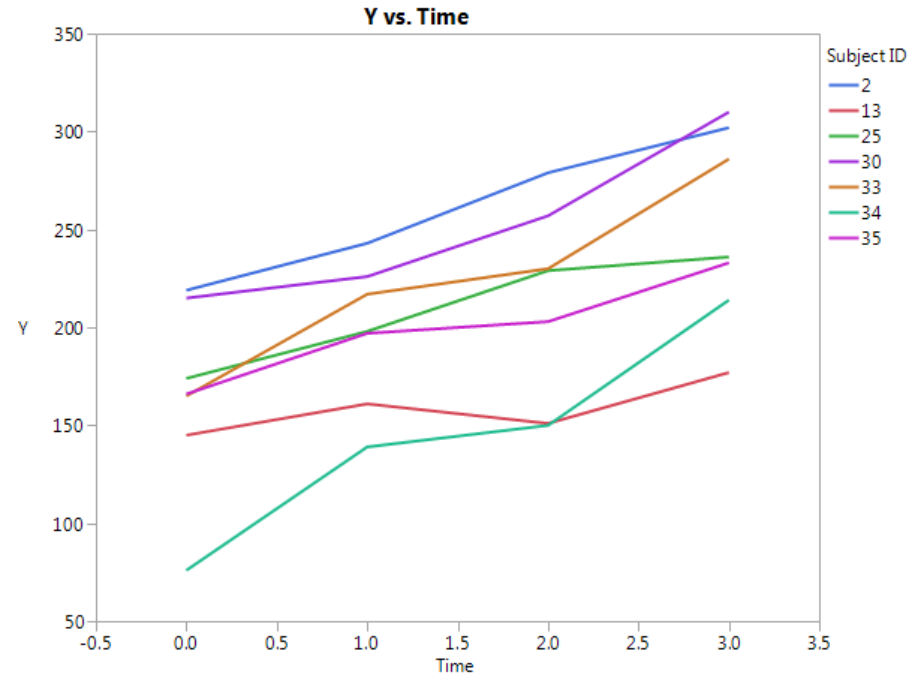
STATISTICS IS A “YOUNG” DISCIPLINE

- Researchers in other disciplines had problems to solve
- Developed similar methods simultaneously
 - R.A. Fisher – biologist, field research at Rothamsted, random effects
 - C.R. Henderson – animal breeding, best linear unbiased predictor (BLUP)
 - A.S. Bryk & S.W. Raudenbush – sociology and education, HLM

DEFINING THE MODEL

A LITTLE MATH

- $Y_{ij} = \beta_0 + b_{0j} + (\beta_1 + b_{1j})X_{ij} + \varepsilon_{ij}$
 - Y_{ij} is the i^{th} observation on the j^{th} subject
 - β_0 and β_1 are the fixed (population) effect intercept and slope
 - b_{0j} and b_{1j} are the random (subject) effect intercept and slope
 - $\begin{pmatrix} b_{0j} \\ b_{1j} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix}\right)$
 - ε_{ij} is the random error for the ij^{th} observation assumed $\sim N(0, \sigma^2)$



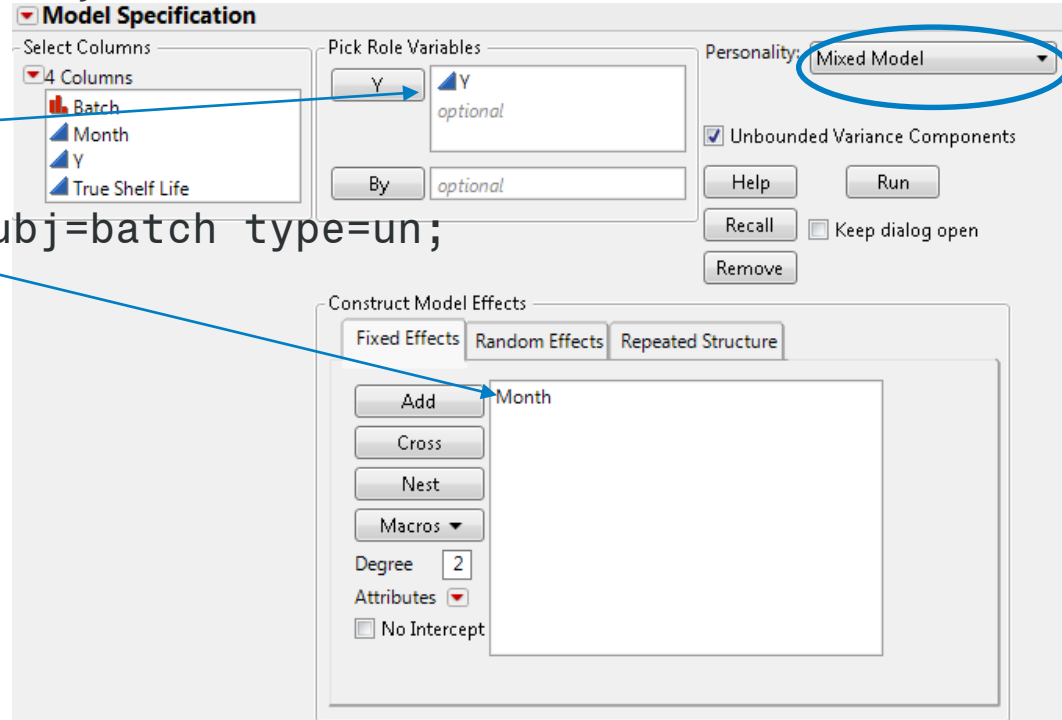
- Singer (1998) showed in a paper “translating” HLM for use with SAS PROC MIXED
- Level 1
 - $Y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + \varepsilon_{ij}$
- Level 2
 - $\beta_{0j} = \beta_{00} + b_{0j} + [\beta_{01}X_{01j} + \dots]$
 - $\beta_{1j} = \beta_{10} + b_{1j} + [\beta_{11}X_{11j} + \dots]$
- Combined
 - $Y_{ij} = \beta_{00} + b_{0j} + (\beta_{10} + b_{1j})X_{ij} + \varepsilon_{ij}$
 - The random coefficient model!

- $Y_{ij} = \beta_{00} + b_{0j} + (\beta_{10} + b_{1j})X_{ij} + \varepsilon_{ij}$
- In SAS
 - ```
proc mixed;
 model y=month;
 random intercept month/subj=batch type=un;
```

- $Y_{ij} = \beta_{00} + b_{0j} + (\beta_{10} + b_{1j})X_{ij} + \varepsilon_{ij}$

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- $Y_{ij} = \beta_{00} + b_{0j} + (\beta_{10} + b_{1j})X_{ij} + \varepsilon_{ij}$
- In SAS
 - `proc mixed;`
`model y=month;`
`random intercept month/subj=batch type=un;`

The screenshot shows the SAS Model Specification dialog box. The 'Pick Role Variables' section has 'Y' selected as the dependent variable and 'By' as the grouping variable. The 'Construct Model Effects' section has 'Random Effects' selected, and the model effects box contains 'Intercept[Batch]&Random Coefficients(1)' and 'Month[Batch]&Random Coefficients(1)'. The 'Degree' is set to 2. Blue arrows point from the SAS code in the list to the corresponding elements in the dialog box.

EXAMPLES

- Pharmaceutical shelf life – using random coefficients and BLUP to determine shelf life
- Education – High School and Beyond hierarchical survey
- Animal growth curves – not limited to linear slopes



THANK YOU!



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