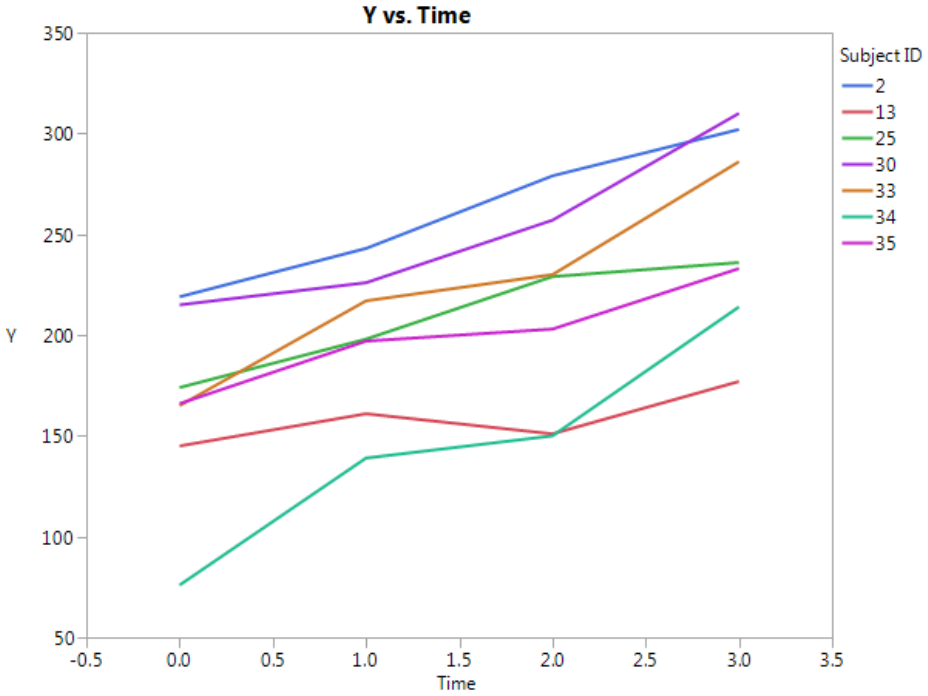
RANDOM COEFFICIENT MODELS

HOW TO MODEL LONGITUDINAL AND HIERARCHICAL DATA IN JMP® PRO

WHAT DO WE MEAN?



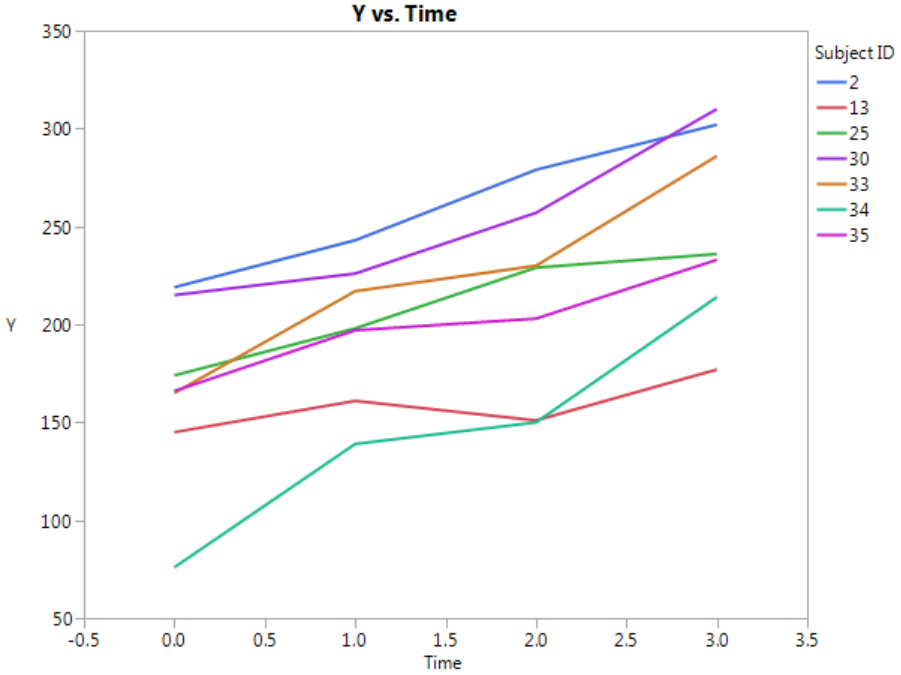
- Intercepts and slopes randomly distributed – potentially correlated
  - Hence “random coefficient”
- Multilevel situations
  - Observe students within schools
  - Commonly referred to as Hierarchical Linear Models (HLM)
- Hierarchical Bayes models
  - Similar in the Bayesian context
  - Not our focus today

STATISTICS IS A “YOUNG” DISCIPLINE

- Researchers in other disciplines had problems to solve
- Developed similar methods simultaneously
  - R.A. Fisher – biologist, field research at Rothamsted, random effects
  - C.R. Henderson – animal breeding, best linear unbiased predictor (BLUP)
  - A.S. Bryk & S.W. Raudenbush – sociology and education, HLM

A LITTLE MATH

- $Y_{ij} = \beta_0 + b_{0j} + (\beta_1 + b_{1j})X_{ij} + \varepsilon_{ij}$
  - Y_{ij} is the i[th] observation on the [jth] subject
  - β_0 and β_1 are the fixed (population) effect intercept and slope
  - b_{0j} and b_{1j} are the random (subject) effect intercept and slope
    - $\begin{pmatrix} b_{0j} \\ b_{1j} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix}\right)$
  - ε_{ij} is the random error for the ij[th] observation assumed ~N(0,σ^2)



A LITTLE MORE MATH

- Singer (1998) showed in a paper “translating” HLM for use with SAS PROC MIXED
- Level 1
  - $Y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + \varepsilon_{ij}$
- Level 2
  - $\beta_{0j} = \beta_{00} + b_{0j} + \left[\beta_{01}X_{01j} + \cdots\right]$
  - $\beta_{1j} = \beta_{10} + b_{1j} + \left[\beta_{11}X_{11j} + \cdots\right]$
- Combined
  - $Y_{ij} = \beta_{00} + b_{0j} + \left(\beta_{10} + b_{1j}\right)X_{ij} + \varepsilon_{ij}$
  - The random coefficient model!

HOW TO FIT

- $Y_{ij} = \beta_{00} + b_{0j} + (\beta_{10} + b_{1j})X_{ij} + \varepsilon_{ij}$
- In SAS
  - ```
    proc mixed;
    model y=month;
    random intercept month/subj=batch type=un;
    ```
    ```

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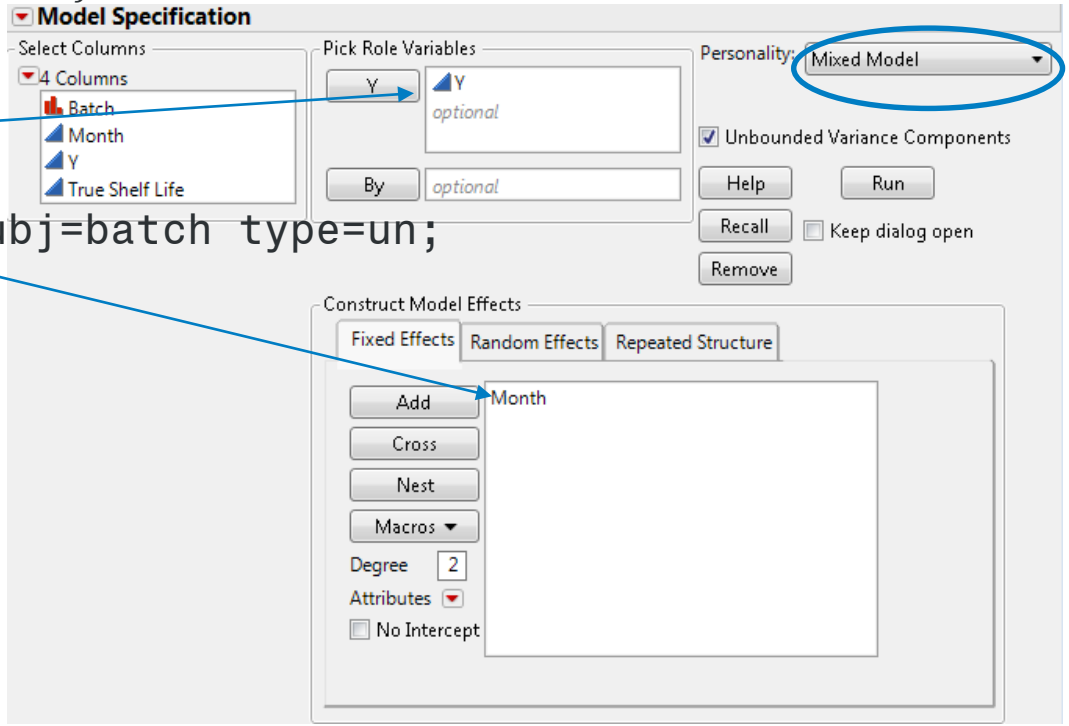

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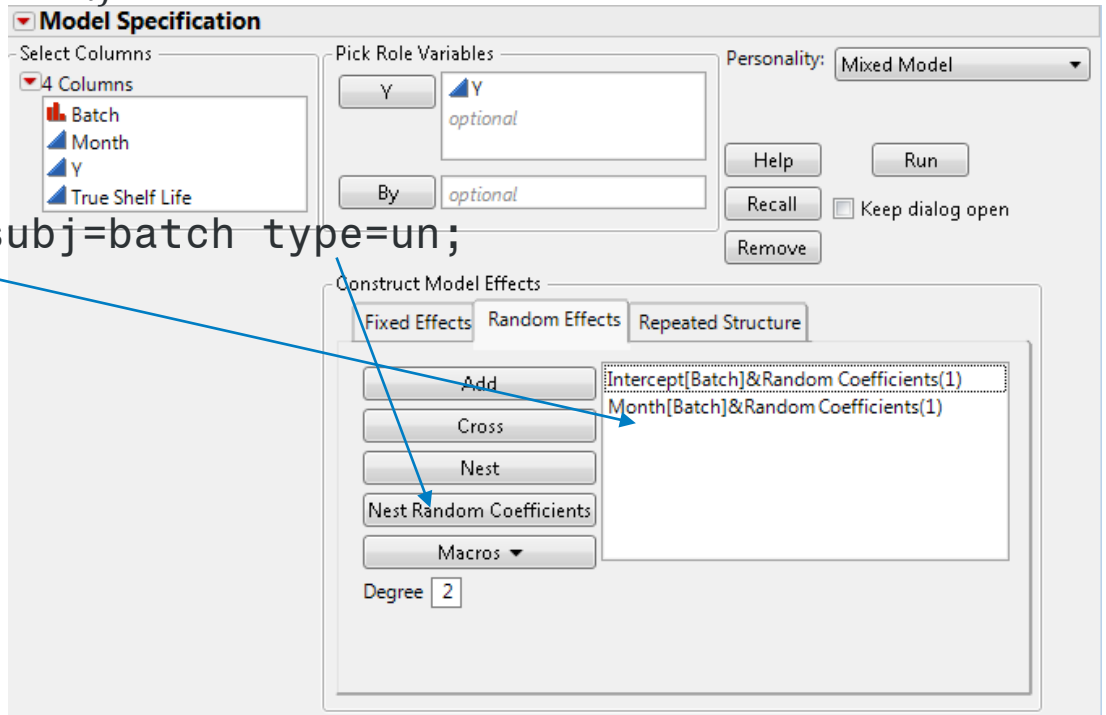

EXAMPLES

- Pharmaceutical shelf life – using random coefficients and BLUP to determine shelf life
- Education – High School and Beyond hierarchical survey
- Animal growth curves – not limited to linear slopes

THANK YOU!