



Statistical Discovery.<sup>TM</sup> From SAS.

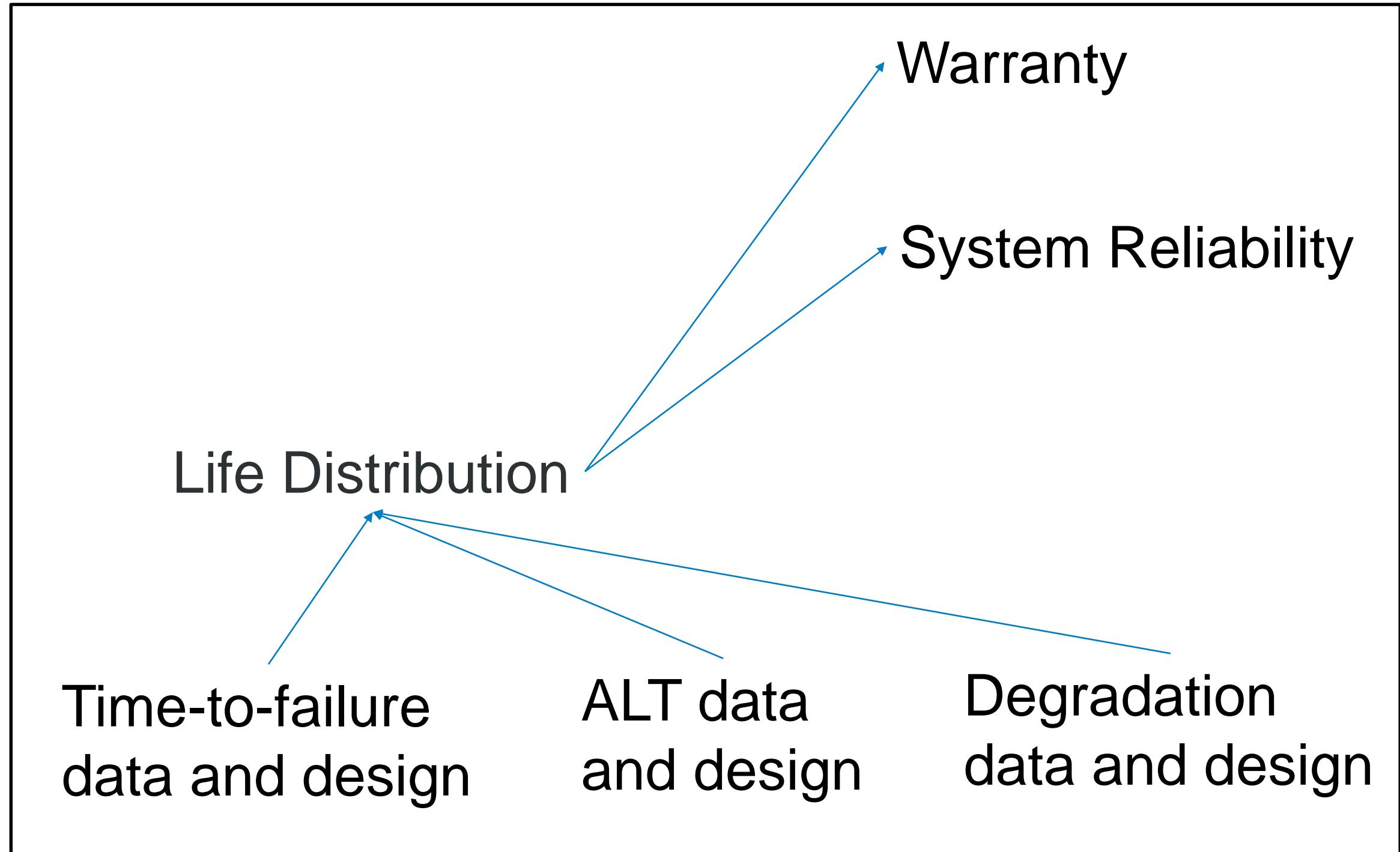
# RELIABILITY ANALYSIS USING **JMP<sup>®</sup>**

Peng Liu  
Leo Wright  
Michael Crotty



# JMP FOR THE NON-REPAIRABLE

- Life Distribution
- Survival
- Reliability Forecast
- Fit Life by X
- Parametric Survival
- Cumulative Damage
- Degradation
- Test Plan (DOE)
- Demonstration Plan (DOE)
- ALT Design (DOE)
- Reliability Block Diagram



# JMP FOR THE REPAIRABLE

- Recurrence Analysis
- Reliability Growth
- Repairable Systems Simulation

Poisson Process

Life Distribution

Reliability Block Diagram

Event Action Sub-diagram

# THIS PRESENTATION

- Life Distribution
- Reliability Forecast
- Repairable Systems Simulation
- Degradation

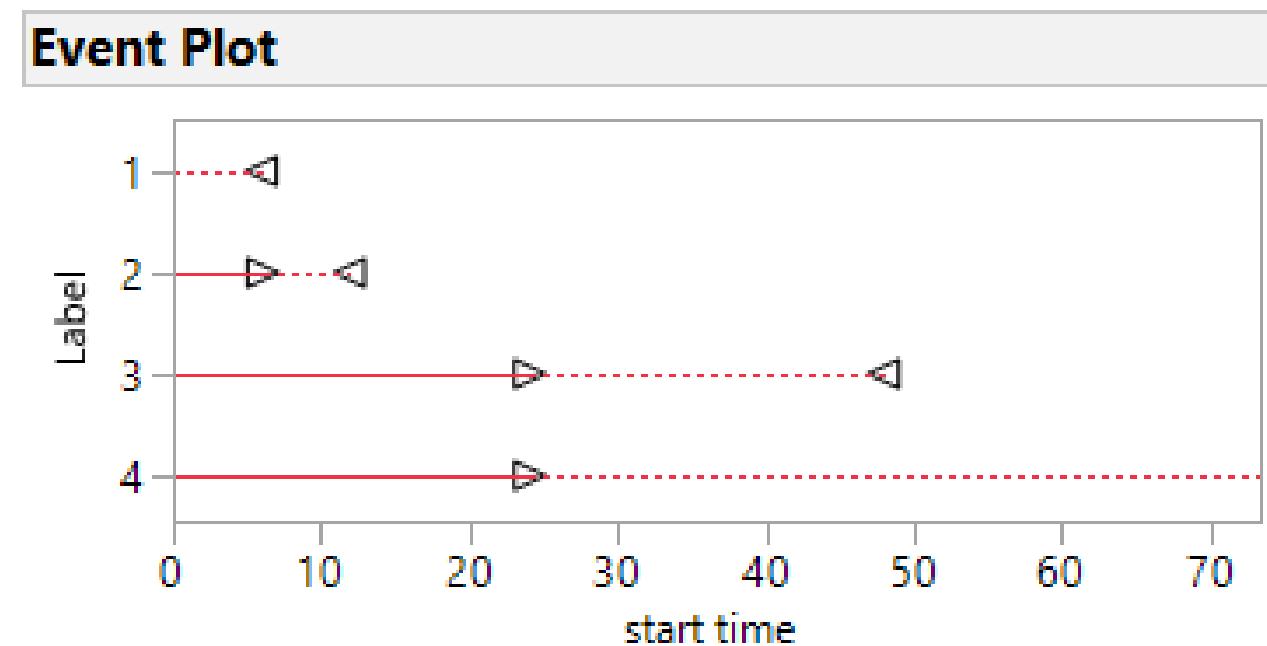
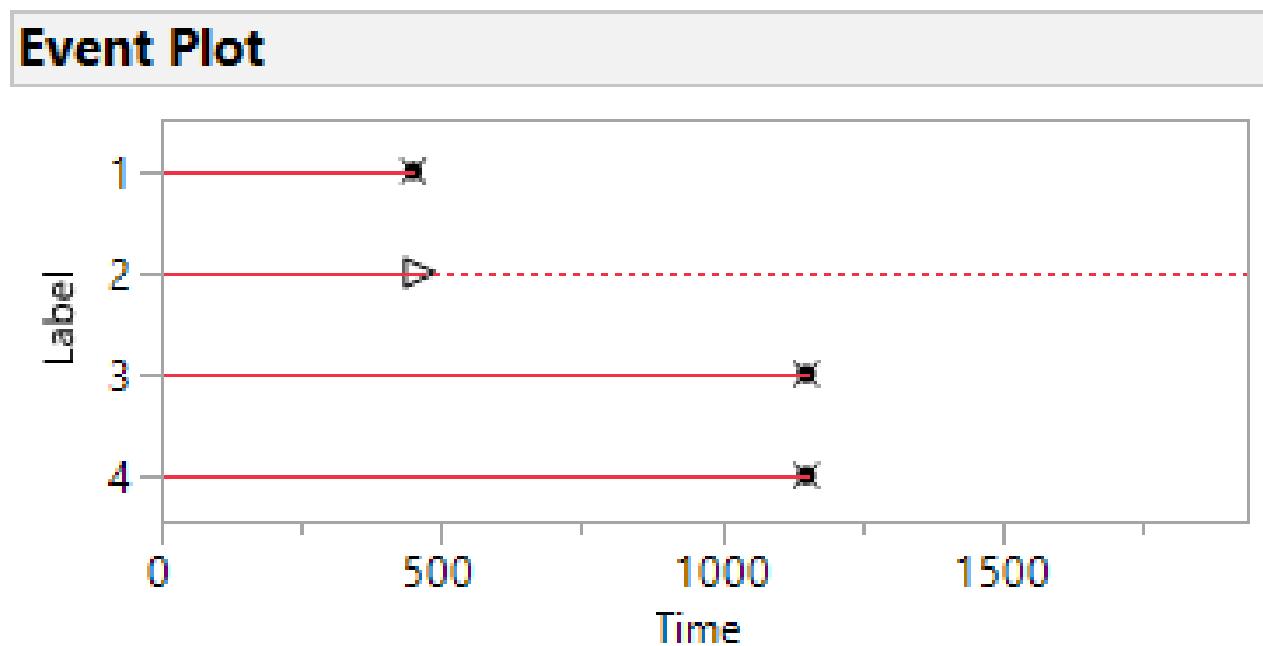
# LIFE DISTRIBUTION

- Purpose of the Analysis:
  - Predict probabilities, percentiles, etc.
- Design Goal of the Software :
  - Fit distributions
  - Compare models
  - Calculate predictions
- Data: Regular data, Censored data, Competing cause
- Distributions:
  - Regular: Weibull, Lognormal, Normal, Logistic, etc.
  - Exotic: Zero inflated, Threshold, Defective subpopulation, Mixture, Latent Cause
- Method: Maximum Likelihood, Bayesian, Weibayes
- Intervals: Wald Intervals, Likelihood Intervals

# LIFE DISTRIBUTION CENSORED DATA

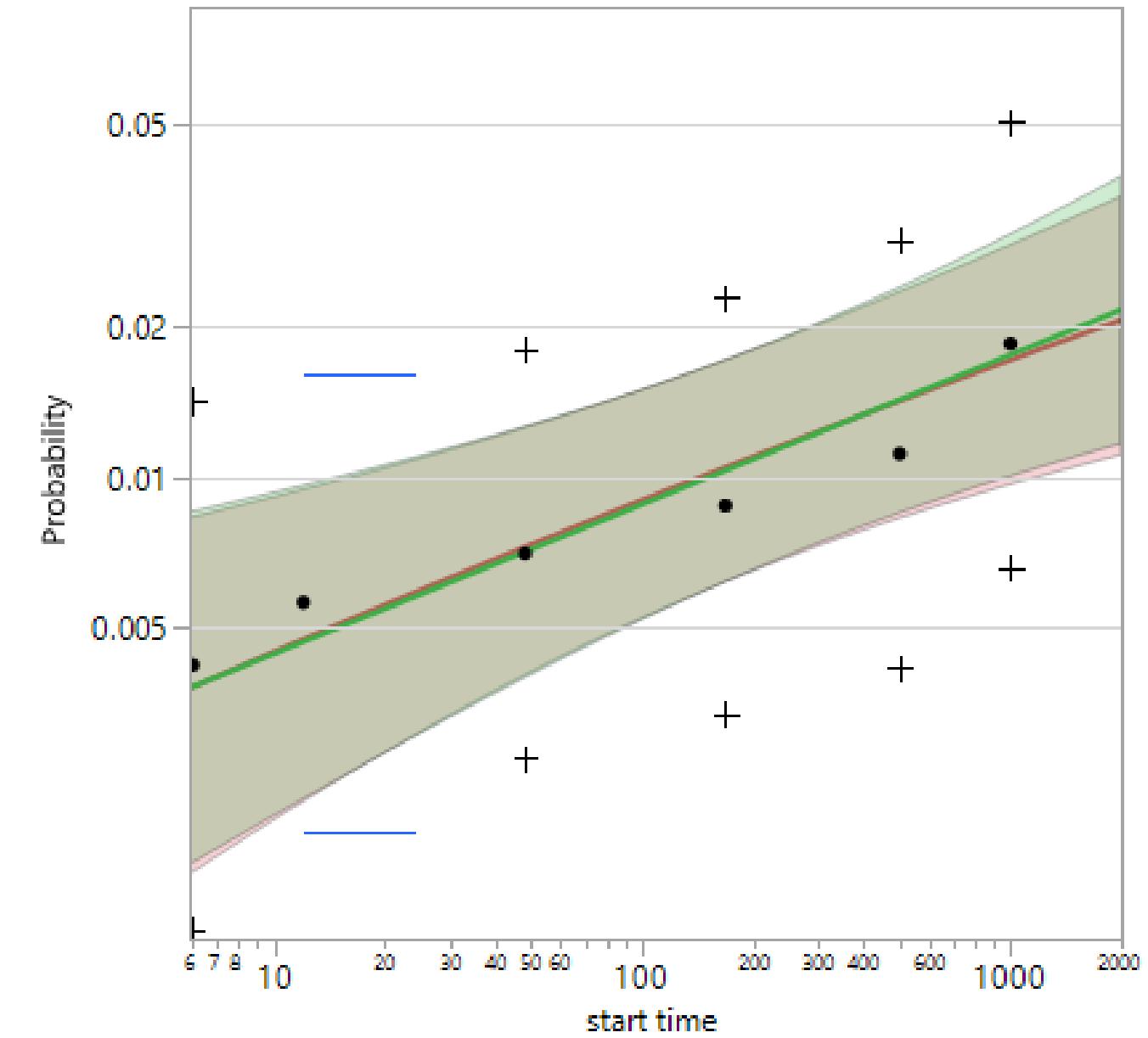
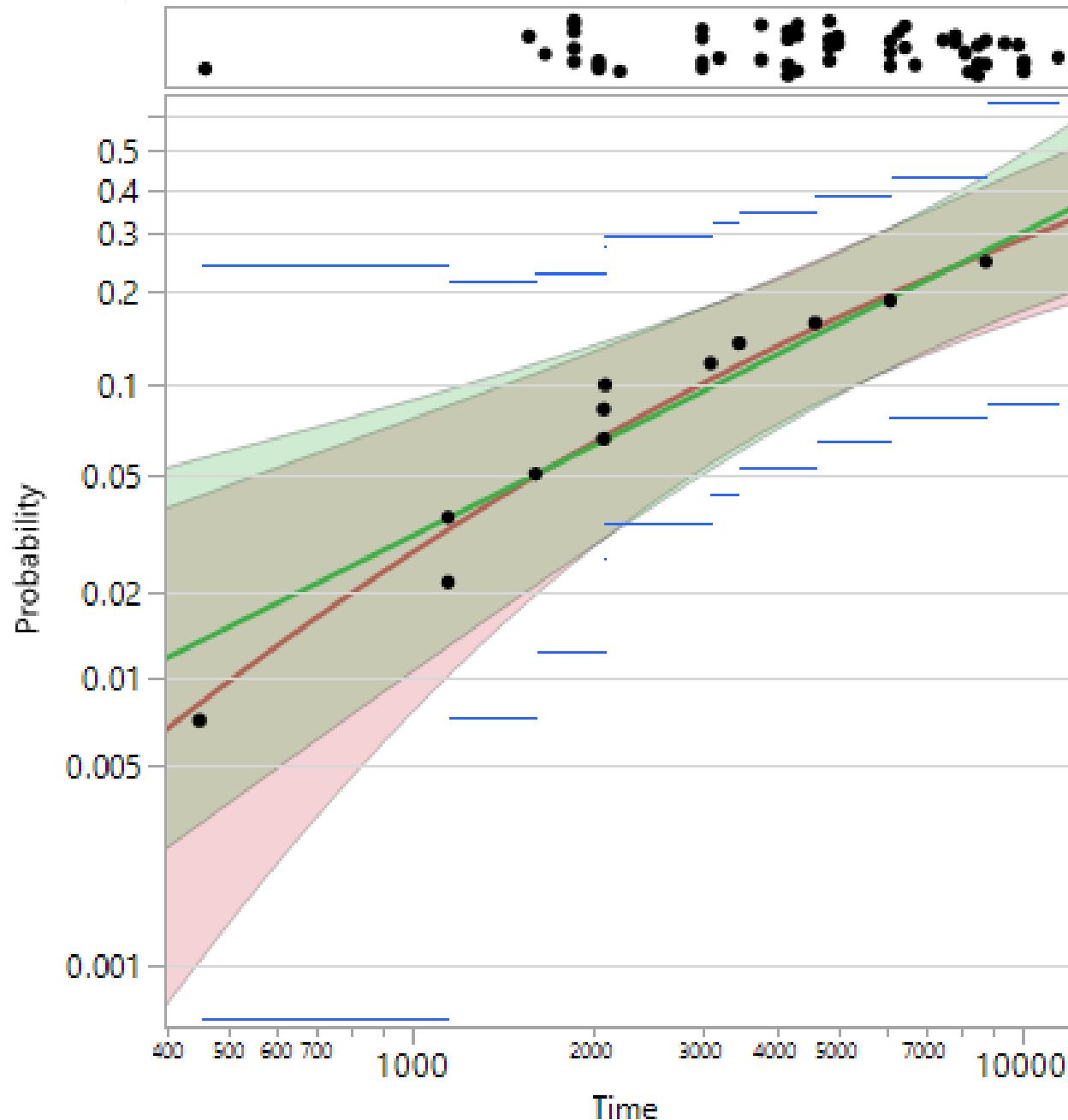
Time	Censor
450	Failed
460	Censored
1150	Failed
1150	Failed

start time	end time
•	6
6	12
24	48
24	•



# LIFE DISTRIBUTION FIND A GOOD FIT (FIT)

Distribution	Scale
<input checked="" type="checkbox"/> Nonparametric	—
<input type="checkbox"/> Lognormal	—
<input checked="" type="checkbox"/> Weibull	—
<input type="checkbox"/> Loglogistic	—
<input type="checkbox"/> Frechet	—
<input type="checkbox"/> Normal	—
<input type="checkbox"/> SEV	—
<input type="checkbox"/> Logistic	—
<input type="checkbox"/> LEV	—
<input type="checkbox"/> Exponential	—
<input type="checkbox"/> LogGenGamma	—
<input type="checkbox"/> GenGamma	—
<input type="checkbox"/> TH Lognormal	—
<input type="checkbox"/> TH Weibull	—
<input type="checkbox"/> TH Loglogistic	—
<input type="checkbox"/> TH Frechet	—
<input type="checkbox"/> DS Lognormal	—
<input type="checkbox"/> DS Weibull	—
<input type="checkbox"/> DS Loglogistic	—
<input type="checkbox"/> DS Frechet	—



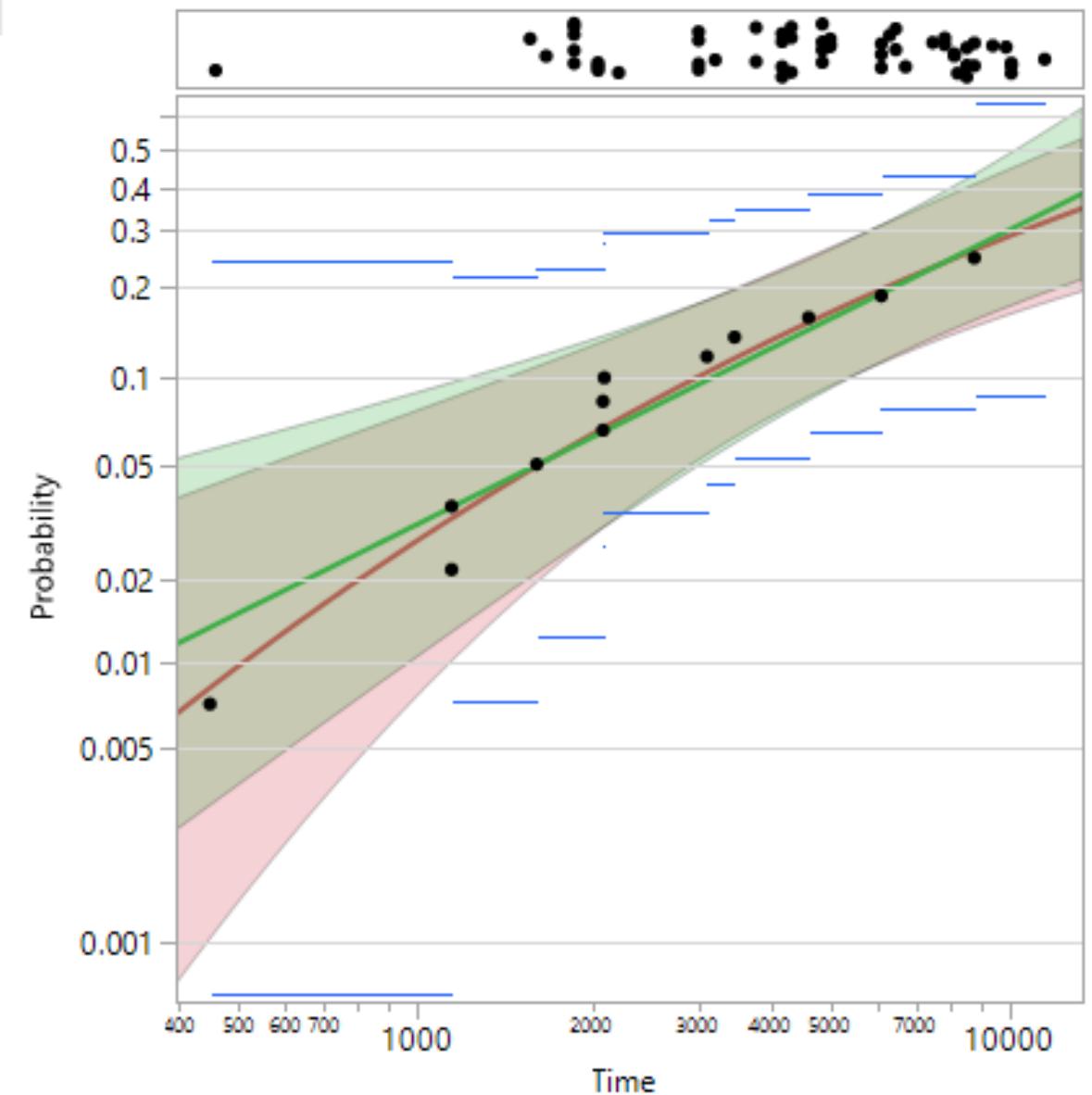
Time	Censor
450	Failed
460	Censored
1150	Failed
1150	Failed

start time	end time
•	6
6	12
24	48
24	•

# LIFE DISTRIBUTION FIND A GOOD FIT (COMPARE & SELECT)

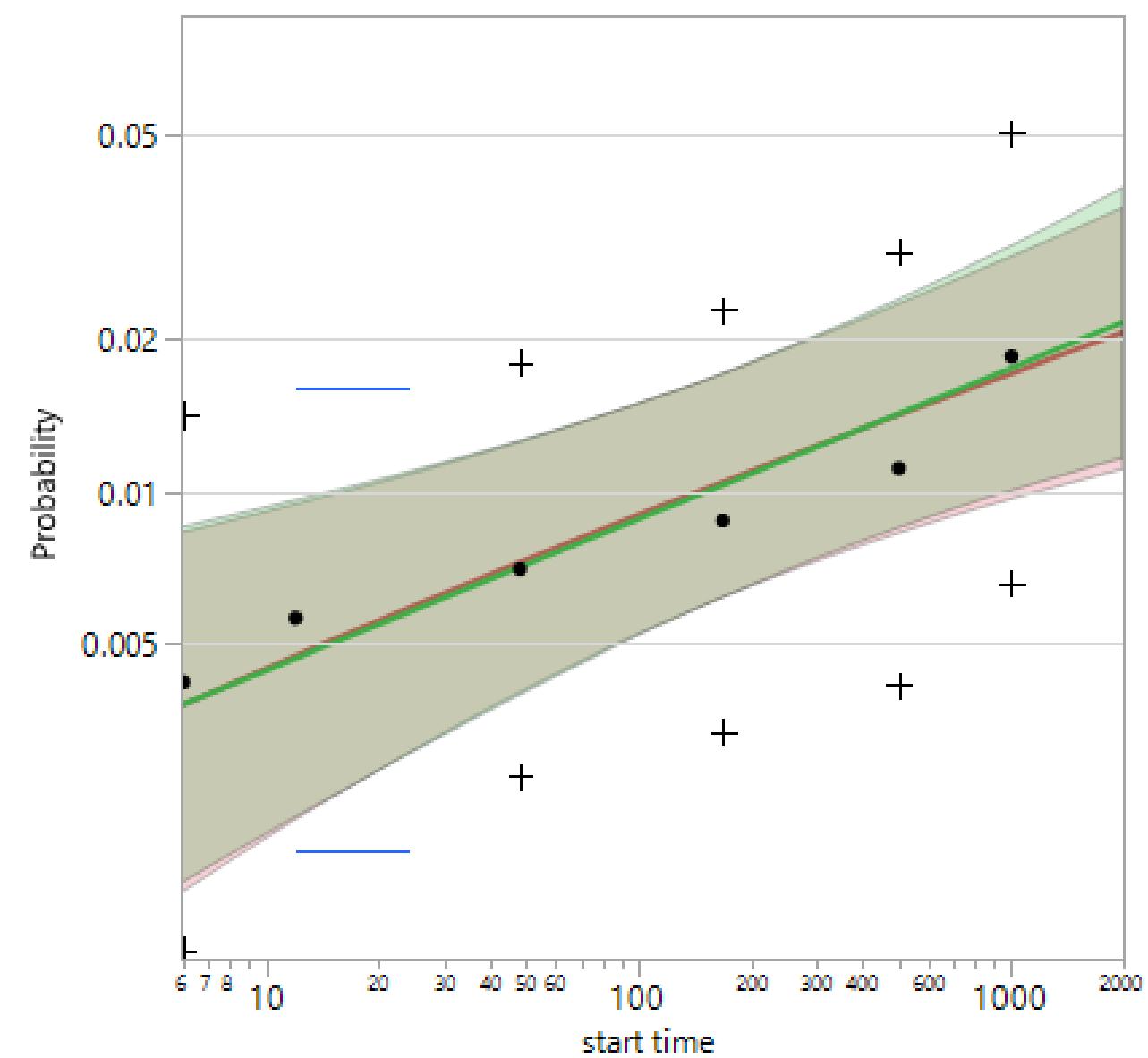
## Compare Distributions

Distribution	Scale
<input checked="" type="checkbox"/> Nonparametric	<input type="radio"/>
<input checked="" type="checkbox"/> Lognormal	<input type="radio"/>
<input checked="" type="checkbox"/> Weibull	<input checked="" type="radio"/>
<input type="checkbox"/> Loglogistic	<input type="radio"/>
<input type="checkbox"/> Frechet	<input type="radio"/>
<input type="checkbox"/> Normal	<input type="radio"/>
<input type="checkbox"/> SEV	<input type="radio"/>
<input type="checkbox"/> Logistic	<input type="radio"/>
<input type="checkbox"/> LEV	<input type="radio"/>
<input type="checkbox"/> Exponential	<input type="radio"/>
<input type="checkbox"/> LogGenGamma	<input type="radio"/>
<input type="checkbox"/> GenGamma	<input type="radio"/>
<input type="checkbox"/> TH Lognormal	<input type="radio"/>
<input type="checkbox"/> TH Weibull	<input type="radio"/>
<input type="checkbox"/> TH Loglogistic	<input type="radio"/>
<input type="checkbox"/> TH Frechet	<input type="radio"/>
<input type="checkbox"/> DS Lognormal	<input type="radio"/>
<input type="checkbox"/> DS Weibull	<input type="radio"/>
<input type="checkbox"/> DS Loglogistic	<input type="radio"/>
<input type="checkbox"/> DS Frechet	<input type="radio"/>



## Compare Distributions

Distribution	Scale
<input checked="" type="checkbox"/> Nonparametric	<input type="radio"/>
<input checked="" type="checkbox"/> Lognormal	<input type="radio"/>
<input checked="" type="checkbox"/> Weibull	<input checked="" type="radio"/>
<input type="checkbox"/> Loglogistic	<input type="radio"/>
<input type="checkbox"/> Frechet	<input type="radio"/>
<input type="checkbox"/> Normal	<input type="radio"/>
<input type="checkbox"/> SEV	<input type="radio"/>
<input type="checkbox"/> Logistic	<input type="radio"/>
<input type="checkbox"/> LEV	<input type="radio"/>
<input type="checkbox"/> Exponential	<input type="radio"/>
<input type="checkbox"/> LogGenGamma	<input type="radio"/>
<input type="checkbox"/> GenGamma	<input type="radio"/>
<input type="checkbox"/> TH Lognormal	<input type="radio"/>
<input type="checkbox"/> TH Weibull	<input type="radio"/>
<input type="checkbox"/> TH Loglogistic	<input type="radio"/>
<input type="checkbox"/> TH Frechet	<input type="radio"/>
<input type="checkbox"/> DS Lognormal	<input type="radio"/>
<input type="checkbox"/> DS Weibull	<input type="radio"/>
<input type="checkbox"/> DS Loglogistic	<input type="radio"/>
<input type="checkbox"/> DS Frechet	<input type="radio"/>



## Model Comparisons

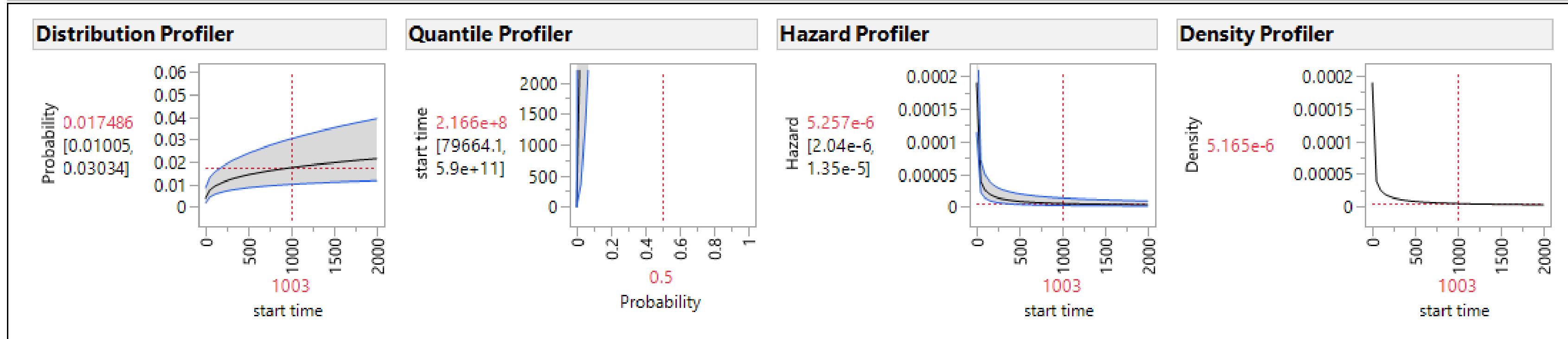
Distribution	AICc	-2Loglikelihood	BIC
Exponential	272.41327	270.35444	274.60294
Frechet	272.70929	268.53019	277.02718
Lognormal	273.27840	269.09930	277.59629
Loglogistic	274.19585	270.01675	278.51374
Weibull	274.48454	270.30544	278.80243

## Model Comparisons

Distribution	AICc	-2Loglikelihood	BIC
Weibull	211.84567	207.83722	222.35827
Loglogistic	211.86332	207.85487	222.37591
Lognormal	212.25011	208.24166	222.76270
Frechet	212.56611	208.55766	223.07870
Exponential	245.19683	243.19402	250.45454

# LIFE DISTRIBUTION PREDICT

## Profilers



### Estimate Probability

start time	Failure Probability	Failure Probability Lower 95% (Wald)	Failure Probability Upper 95% (Wald)	Failure Probability Lower 95% (Likelihood)	Failure Probability Upper 95% (Likelihood)
1000.0000	0.01747044	0.01004375	0.03030396	0.00966549	0.02933600
2000.0000	0.02144855	0.01164662	0.03933412	0.01133987	0.03847784



### Estimate Quantile

Failure Probability	Life Time Quantile	Life Time Quantile Lower 95% (Wald)	Life Time Quantile Upper 95% (Wald)	Life Time Quantile Lower 95% (Likelihood)	Life Time Quantile Upper 95% (Likelihood)
0.10000000	396456.93	4433.3901	35453253	18598.694	682815289
0.20000000	4882210.85	14310.6118	1665615916	92149.984	7.7069e+10



## LIFE DISTRIBUTION STATISTICAL DETAILS

### Methods:

- Maximum Likelihood (default)
- Bayesian (available for regular distributions)
- Weibayes (when there are no failures; available in Weibull reports)

### Interval Types:

- Wald (when there are sufficient failures)
- Likelihood (otherwise)

Note: Intervals for parameter estimates and intervals for other predictions.

# LIFE DISTRIBUTION EXAMPLE

# RELIABILITY FORECAST

- Purpose of the Analysis:
  - Predict failure counts in the future:  $N_t = \sum n_{t,i}$
  - Make a statement about the uncertainty in the prediction:  $(\underline{N}_t, \overline{N}_t)$
- Design Goal of the Software:
  - Explore raw data, by batch, group, and time.
  - Fit life distributions and compare
  - Arrange risk sets, produce forecast
- Data:
  - Nevada format, Dates format, Time-to-Event format
- Forecasting Types: Incremental; Cumulative
- Forecasting Intervals: Plug-in intervals; Prediction intervals

# RELIABILITY FORECASTING

Sold Quantity	Sold Month	8/09	9/09	10/09	11/09	12/09	1/10	2/10	At Risk	3/10	4/10	...	
2550	7/09	11	13	25	24	33	18	55	2371	$p_{1,1}$	$p_{1,2}$		
2600	8/09		8	19	30	30	29	29	2455	$p_{2,1}$	$p_{2,2}$		
2650	9/09			14	18	25	26	27	2540	$p_{3,1}$	$p_{3,2}$		
2700	10/09				13	17	34	33	2603	$p_{4,1}$	$p_{4,2}$		
2750	11/09					12	21	29	2688	$p_{5,1}$	$p_{5,2}$		
2800	12/09						6	16	2778	$p_{6,1}$	$p_{6,2}$		
2850	1/10							17	2833	$p_{7,1}$	$p_{7,2}$		

$$N_t = \sum_{batch} N_{batch, t}$$

# UNCERTAINTIES IN FORECASTING

## Uncertainty Sources

- Knowing outcome probability, the outcome is still random, e.g. toss a coin.
- Probabilities might be estimated. Themselves involve uncertainties.

## Solutions

- Uncertainties can be quantified using intervals.

## Option Chart

	No Intervals	Plugin Intervals	Prediction Intervals
Incremental Counts		Ignore parameter uncertainty	Consider parameter uncertainty
Cumulative Counts		Ignore parameter uncertainty	Consider parameter uncertainty
	<input checked="" type="checkbox"/> Approximate Binomial by Poisson		

# RELIABILITY FORECASTING

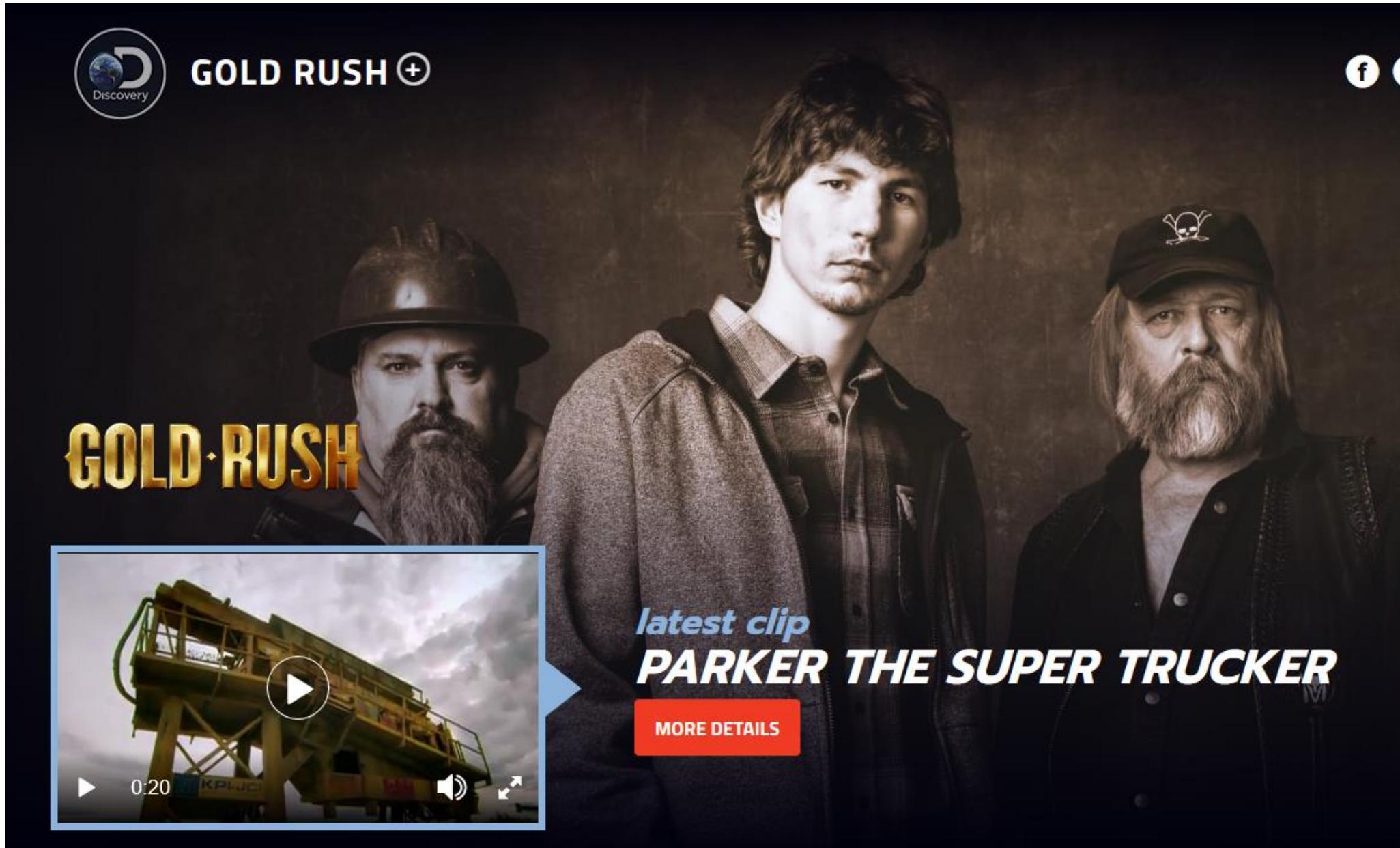
## EXAMPLE

# **REPAIRABLE SYSTEMS SIMULATION**

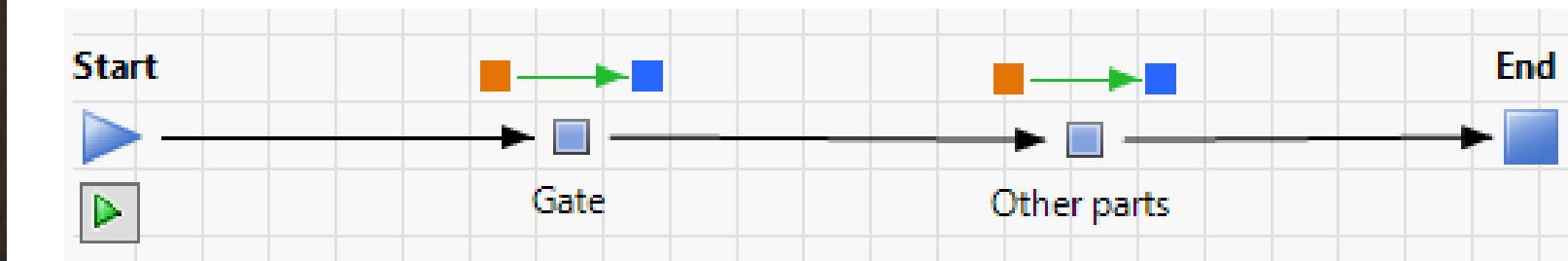
- For studying:
  - System reliability
  - When and how to repair
- Design Goals of the Software:
  - Expressive
  - Customizable
- System representation: Reliability Block Diagram
- Repair representation: Event-Action Sub-diagram
- Simulate: Events and changes of failure probabilities
- Collect: Events (times and consequences)

# REPAIRABLE SYSTEMS SIMULATION

## WHAT DOES IT SIMULATE?



7/2/2017 screenshot



# **REPAIRABLE SYSTEMS SIMULATION**



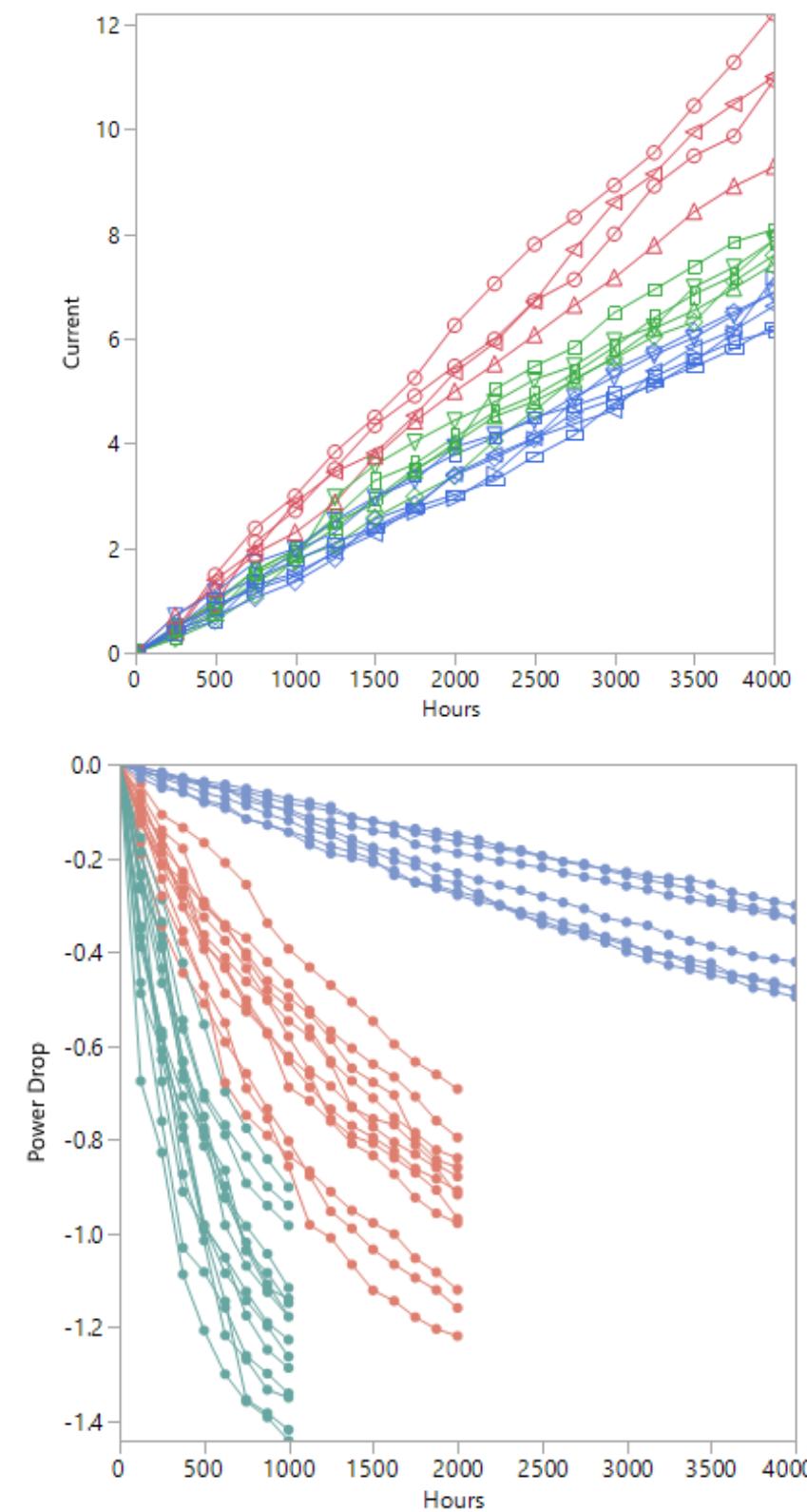
Conditions: 4 identical pumps, 1 different pump

Alternative 1: Replace the old with new

Alternative 2: Add one new pump

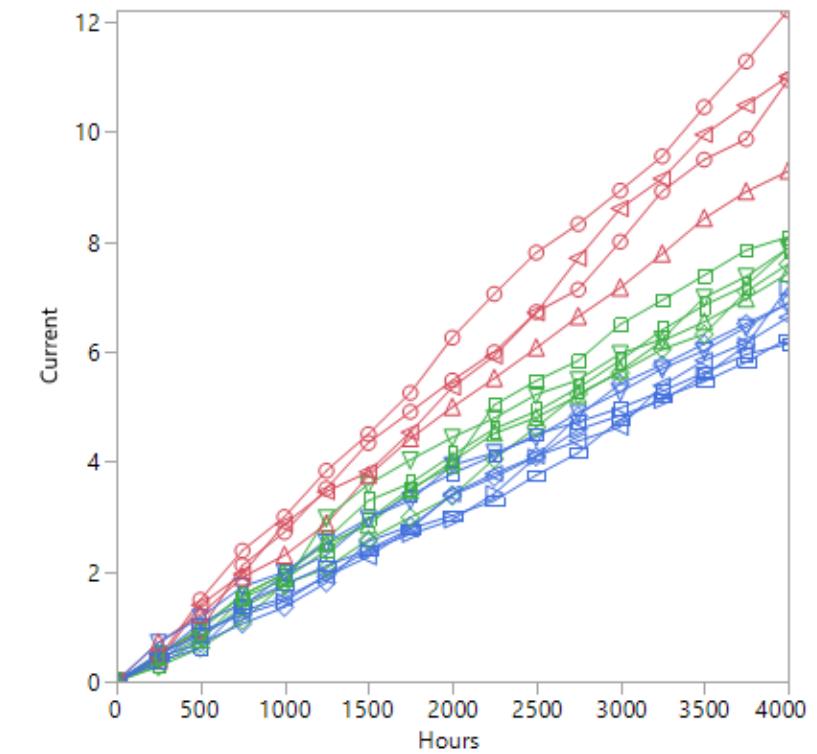
# DEGRADATION

- Purposes of the Analysis:
  - Find pseudo failure times
  - Study kinetics
- Design Goals of the Software:
  - Fit built-in models automatically
  - Provide a user interface for fitting custom models
- Method: Least squares
- Path Types: Linear; Nonlinear
- Built-in Models
- Custom Models
  - Model building round trips



# DEGRADATION BUILT-IN LINEAR MODELS

- Linear path models:  $f(Y) \sim N(\mu = \beta_0 + \beta_1 g(t), \sigma)$
- $f$  and  $g$  are transform functions, e.g.  $\ln$ ,  $\exp$ ,  $\sqrt{t}$ , etc.



		Intercept			
$\mu =$		Zero	Common	Common in Group	Different
Slope	Common	$\beta_1 g(t)$	$\beta_0 + \beta_1 g(t)$	$\beta_{0,k} + \beta_1 g(t)$	$\beta_{0,b} + \beta_1 g(t)$
	Common in Group	$\beta_{1,k} g(t)$	$\beta_0 + \beta_{1,k} g(t)$	$\beta_{0,k} + \beta_{1,k} g(t)$	$\beta_{0,b} + \beta_{1,k} g(t)$
	Different	$\beta_{1,b} g(t)$	$\beta_0 + \beta_{1,b} g(t)$	$\beta_{0,k} + \beta_{1,b} g(t)$	$\beta_{0,b} + \beta_{1,b} g(t)$

Subscript  $k$  denotes the group ID;  $b$  denotes batch ID.

## DEGRADATION BUILT-IN NONLINEAR MODELS

- Nonlinear path models:  $Y \sim N(\mu = h(t, \theta, X), \sigma)$
- $h$  is specified via a JSL parameter function.
  - Reaction Rate:  $\mu = D_\infty(1 - e^{-R_U \times AF(Temp) \times t})$
  - Reaction Rate Type I:  $\mu = D_\infty e^{-R_U \times AF(Temp) \times t}$
  - Constant Rate:  $\mu = m(\beta_0 + g(X) \times f(t))$ 
    - Path transformation:  $m$  can be Identity, *exp*, *log*
    - Rate transformation:  $g$  can be Arrhenius, Power, or *exp*
    - Time transformation  $f$  can be Identity, *sqr*

# DEGRADATION EXAMPLE



Statistical Discovery.<sup>TM</sup> From SAS.

Learn more about JMP<sup>®</sup> at  
[jmp.com](http://jmp.com)

