The Role of Perception in Statistics-Based Decisions

Bryan Fricke

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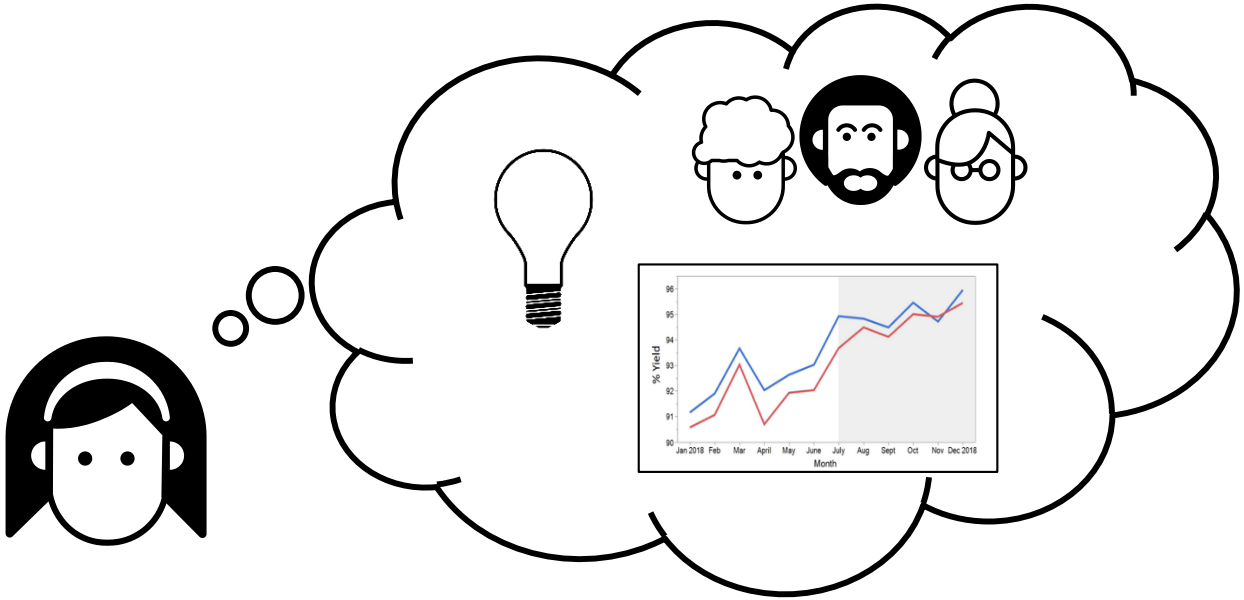
Creating Effective Graphs

1. What question is being addressed?
2. Who is the intended audience?
3. Does the graph communicate effectively?



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Decision Making Under Risk

- Researchers have attempted to understand how people make decisions under uncertainty by examining gambles
- For example, 80% chance to win $100 and 20% chance to win $10

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Expected Value of Gamble

$$E[X] = \sum_{i=1}^{n} p_i x_i$$

For example,

80% chance to win $100

AND

20% chance to win $10

$$E[X] = 0.80 \times \$100 + 0.20 \times \$10 = \$82$$

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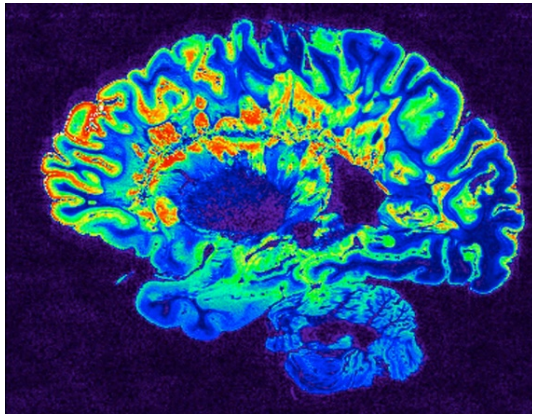
Expected Utility Hypothesis

- Expected Value

  Expected dollar value of a gamble

- Expected Utility

  Expected utility (psychological value) of a gamble



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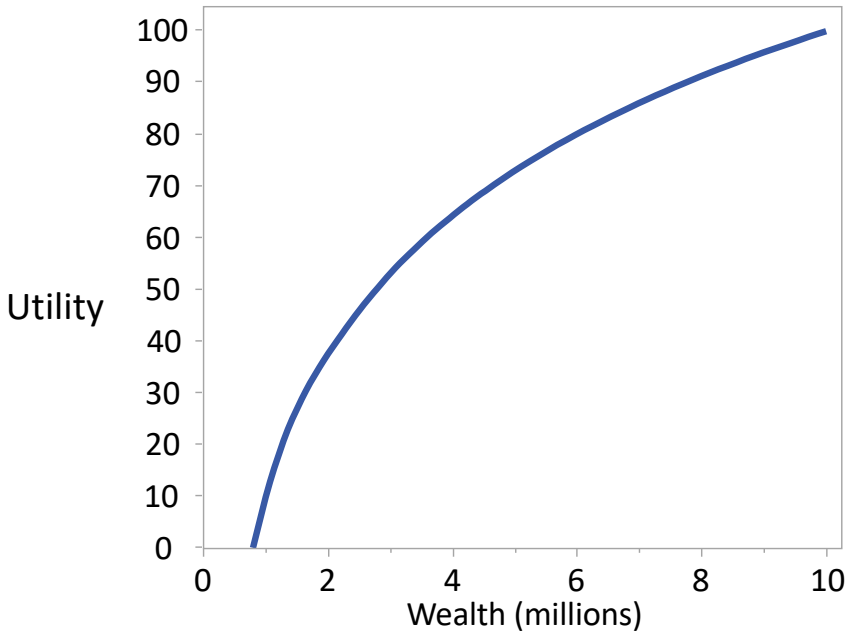
Expected Utility Hypothesis

$$u(w) = k \ln \frac{w}{w_0}$$



D. Bernoulli, *Exposition of a New Theory on the Measurement of Risk,* 1738

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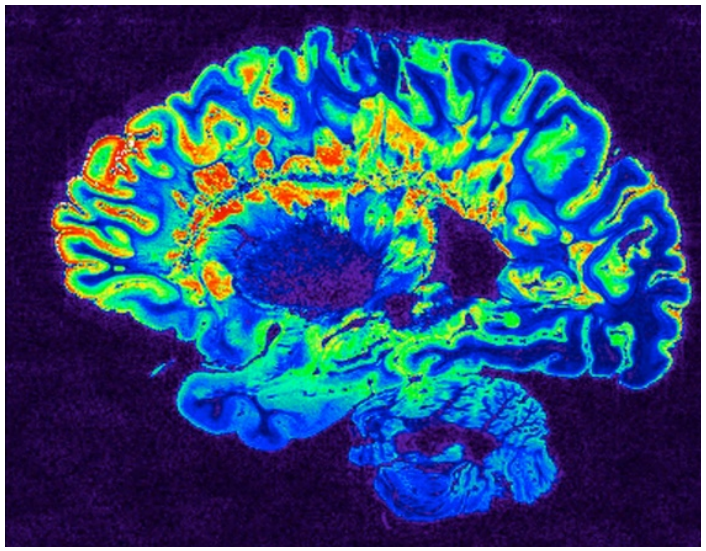
Expected Utility Hypothesis

$$E[U] = \sum_{i=1}^{n} p_i k \ln \frac{w_i}{w_0}$$



D. Bernoulli, *Exposition of a New Theory on the Measurement of Risk,* 1738

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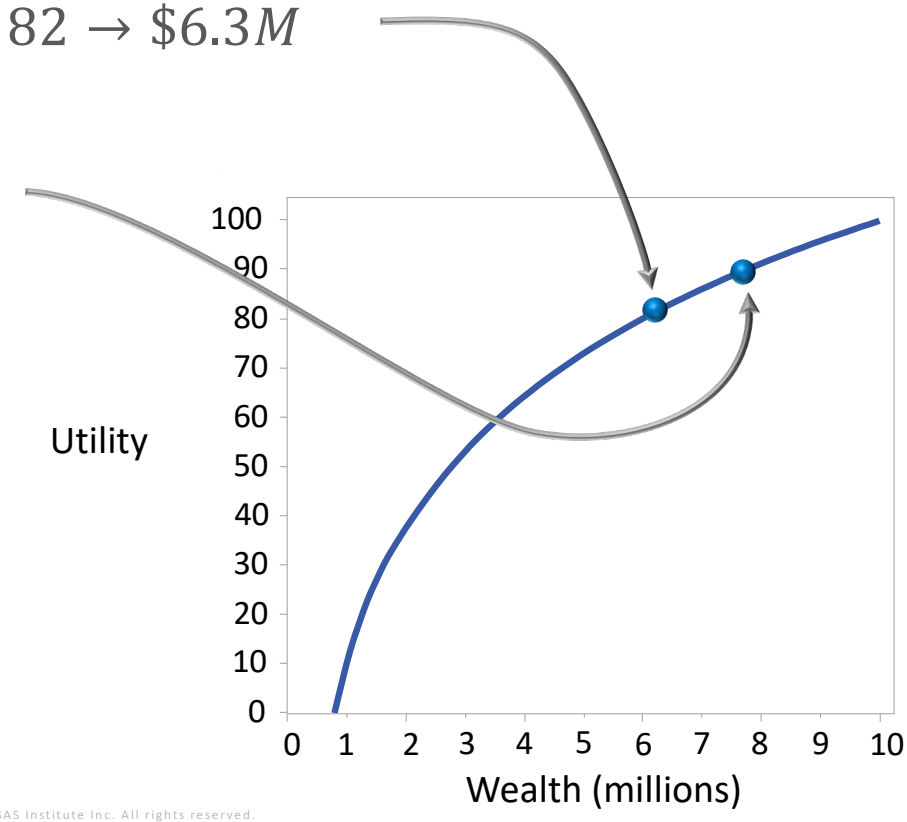
Expected Utility Hypothesis

- 80% chance to win \$10M and 20% chance to win \$1M
  - $E[U] = 0.8 \times 100 + 0.2 \times 10 = 82 \rightarrow \$6.3M$
- \$8M for certain
  - $E[U] = 1.0 \times 91 = 91 \rightarrow \$8M$



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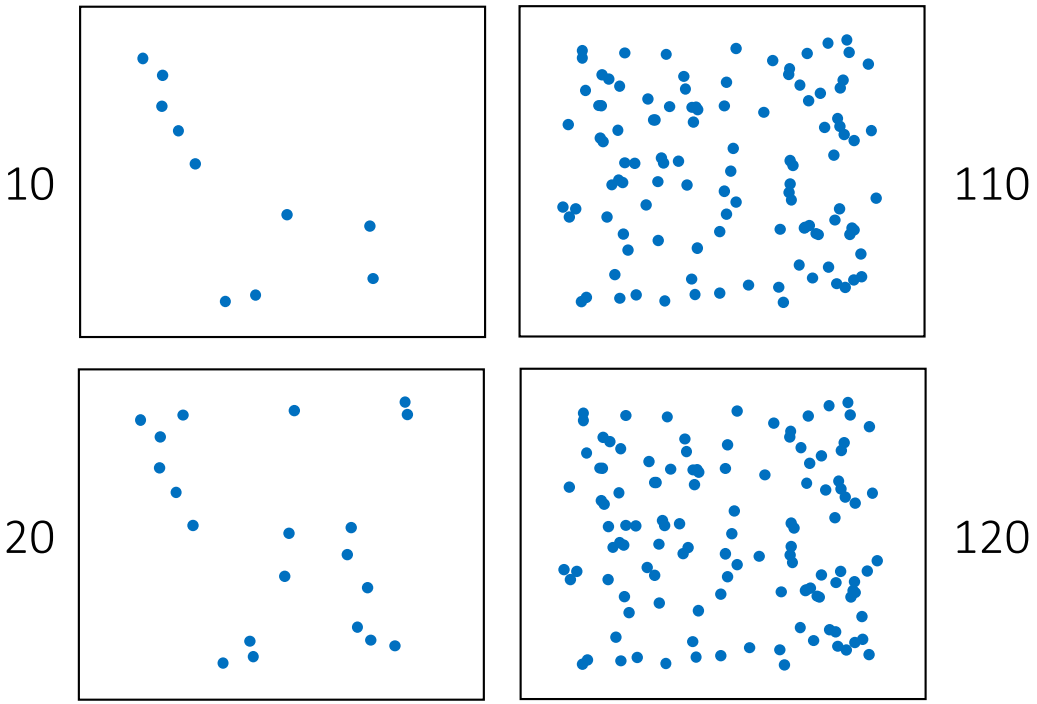
Psychophysics

Fechner's Law

$$p = k \ln \frac{S}{S_0}$$



G. T. Fechner, *Elemente der Psychophysik*, 1860

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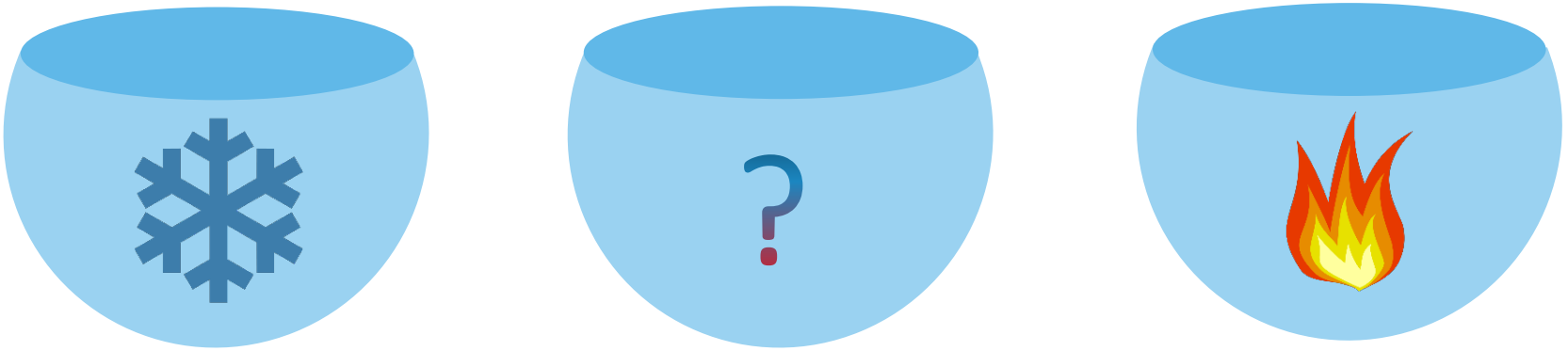
Psychophysics

Adaption Level

- Place one hand in cold water and one hand in warm water for a minute
- Place both hands in middle bowl which has water at room temperature
- How do you perceive the temperature?



H. Helson, *Adaptation-Level as Frame of Reference for Prediction of Psychophysical Data,* 1947

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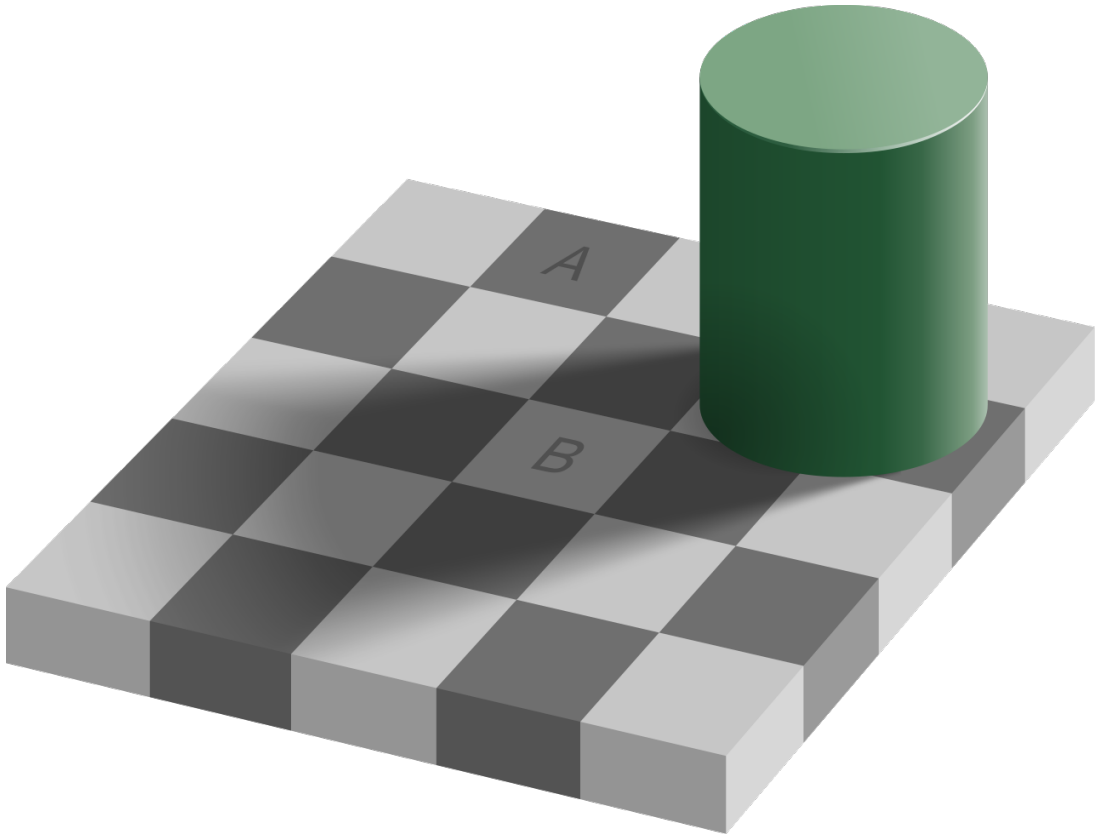
Psychophysics



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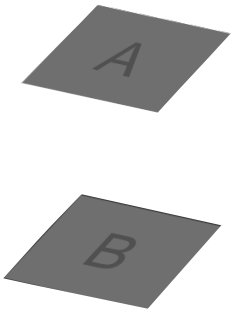
Psychophysics



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Prospect Theory

Three Principles

1. Decreasing sensitivity to changes as both positive and negative amounts increase
2. Psychological value is relative to reference point (adaption level) rather than absolute wealth
3. Losses are more aversive than gains are attractive

Kahneman and Tversky, *Prospect Theory: An Analysis of Decision Under Risk*, 1979

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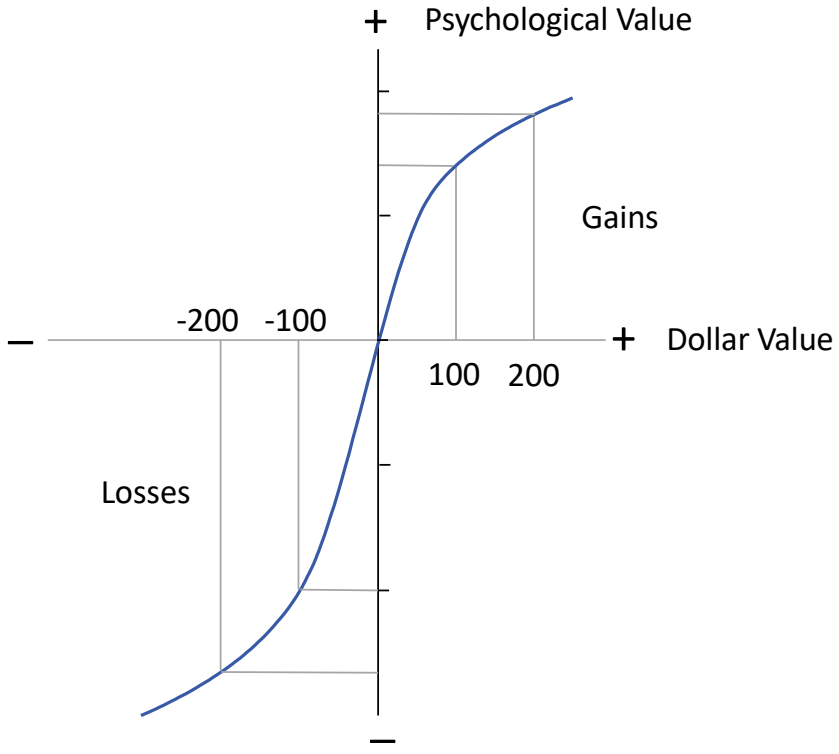
Prospect Theory



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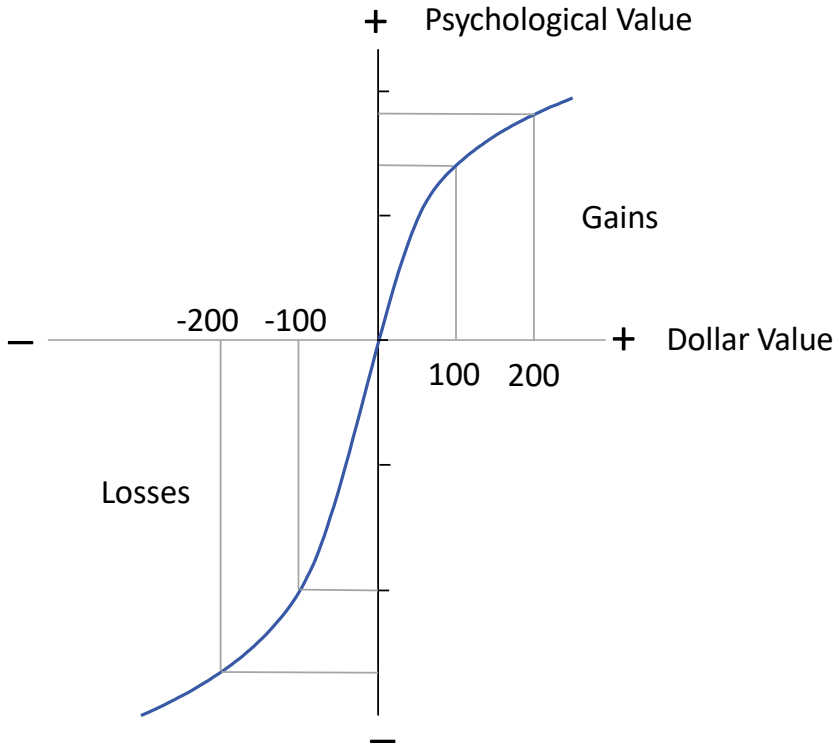
Prospect Theory



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Prospect Theory

Decision Weights

Assume you have some chance of winning $1,000. Do you perceive the following changes in your odds as equal improvements?

- 0 to 5%
- 5% to 10%
- 60% to 65%
- 95% to 100%

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Prospect Theory

Decision Weights

Assume you have some chance of winning $1,000. Do you perceive the following changes in your odds as equal improvements?

- 0 to 5%
  - Possibility effect
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- 60% to 65%
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Prospect Theory

Decision Weights

Assume you have some chance of winning $1,000. Do you perceive the following changes in your odds as equal improvements?

- 0 to 5%
  - Possibility effect
- 5% to 10%
- 60% to 65%
- 95% to 100%
  - Certainty effect

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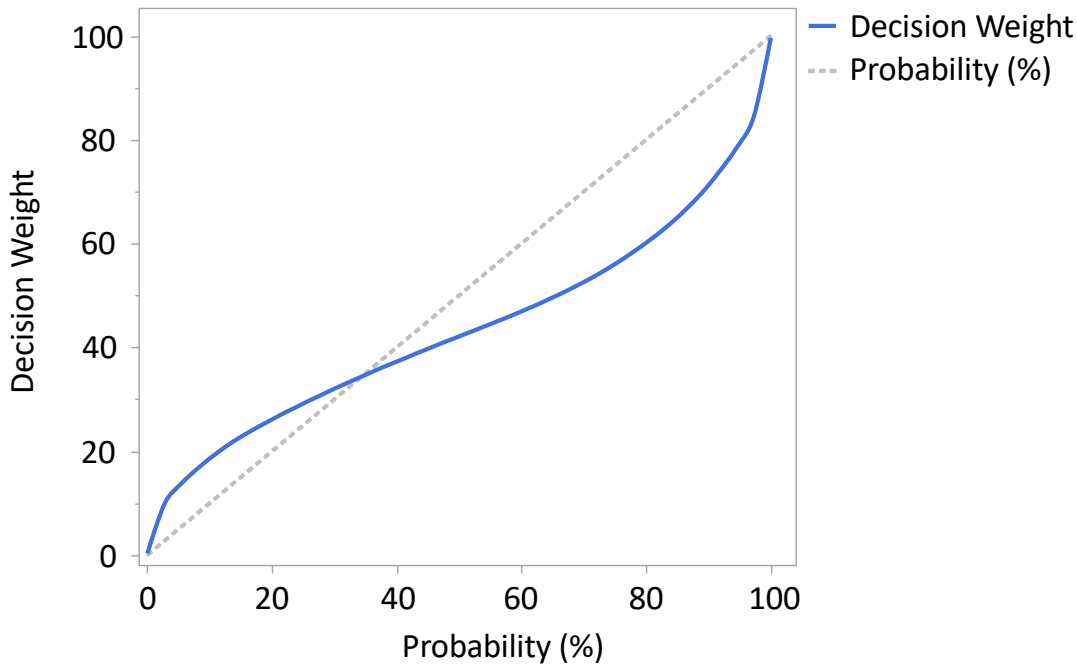
Prospect Theory

Decision Weights



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Prospect Theory

Fourfold Pattern

	GAINS	LOSSES
HIGH PROBABILITY Certainty Effect	95% chance to win $10,000 Fear of disappointment RISK AVERSE	95% chance to lose $10,000 Hope to avoid loss RISK SEEKING
LOW PROBABILITY Possibility Effect	5% chance to win $10,000 Hope of large gain RISK SEEKING	5% chance to lose $10,000 Fear of large loss RISK AVERSE

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Prospect Theory
Endowment Effect

Sellers

$7.12

Choosers

$3.12

Buyers

$2.87

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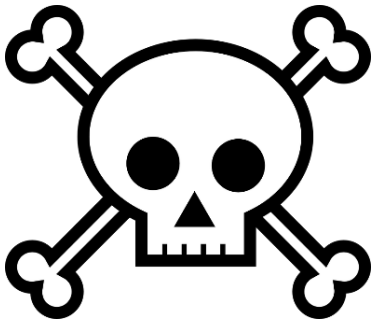
Decision Time

An outbreak of an Asian disease is expected to kill 600 people

- Option 1
  - 400 people will die
- Option 2
  - 1/3 probability no one dies

  AND
  - 2/3 probability that 600 people die



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jmp

Decision Time

An outbreak of an Asian disease is expected to kill 600 people

- Option 1
  - 200 people will be saved
- Option 2
  - 1/3 probability 600 people will be saved

  AND
  - 2/3 probability no one will be saved

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Prospect Theory
Framing Effect

- Positive frame
  - One-month survival rate for surgery is 90%
  - 84% of physicians chose surgery
- Negative frame
  - 10% mortality in the first month after surgery
  - 50% of physicians chose radiation

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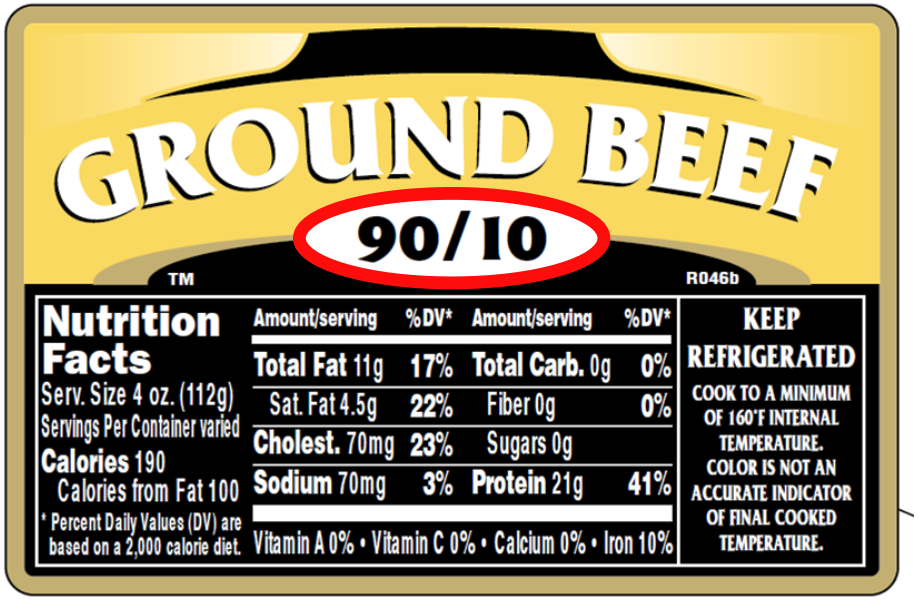
Prospect Theory

Framing Effect

USDA only allows labels such as “90% Lean” if there is another label that displays “10% fat”



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Prospect Theory
Frame of Reference

- Tom trades in a 12 mpg car for a 14 mpg car
- Kim trades in a 30 mpg car for a 40 mpg car
- Assuming both drive 12K miles per year, who will save more gas by switching to cars?

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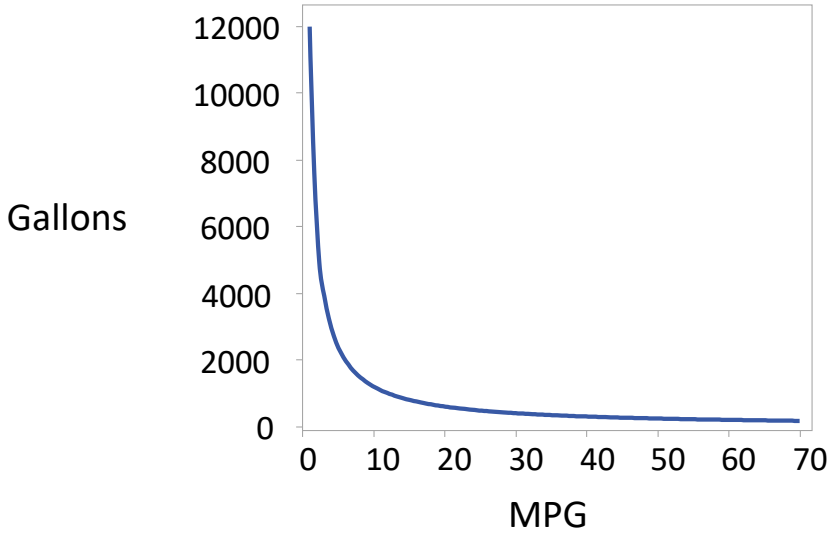
Prospect Theory
Frame of Reference

- Gallons used is inversely proportional to mpg
- So, given a fixed number of miles driven, gallons becomes increasingly insensitive to changes in mpg as mpg increases



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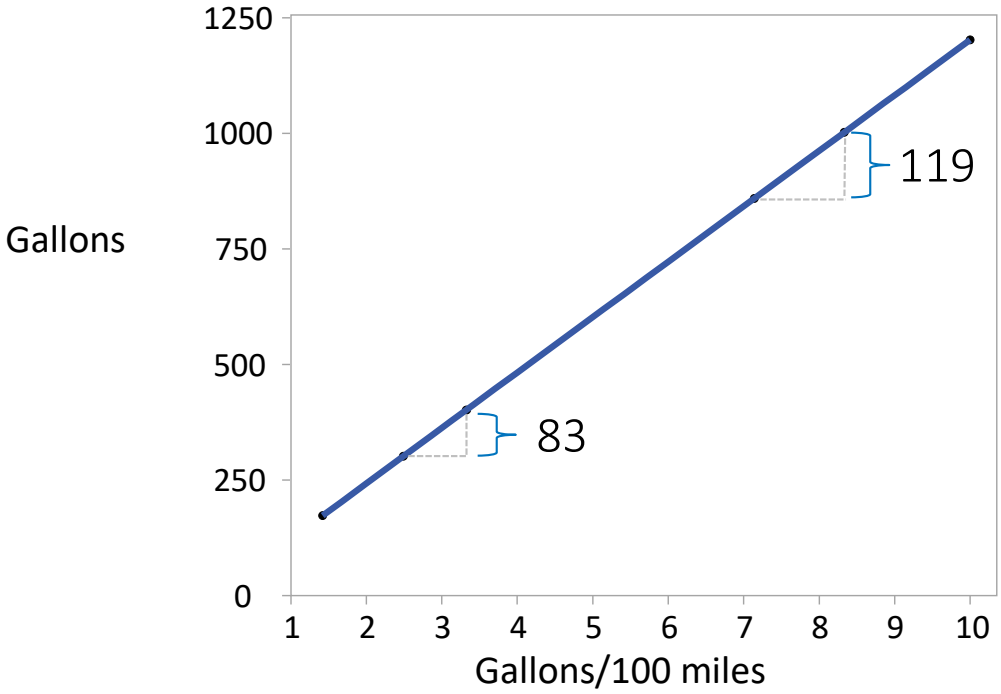
Prospect Theory
Frame of Reference

- Tom, switching from 12 to 14 mpg, saves 119 gallons
- Kim, switching from 30 to 40 mpg, saves 83 gallons



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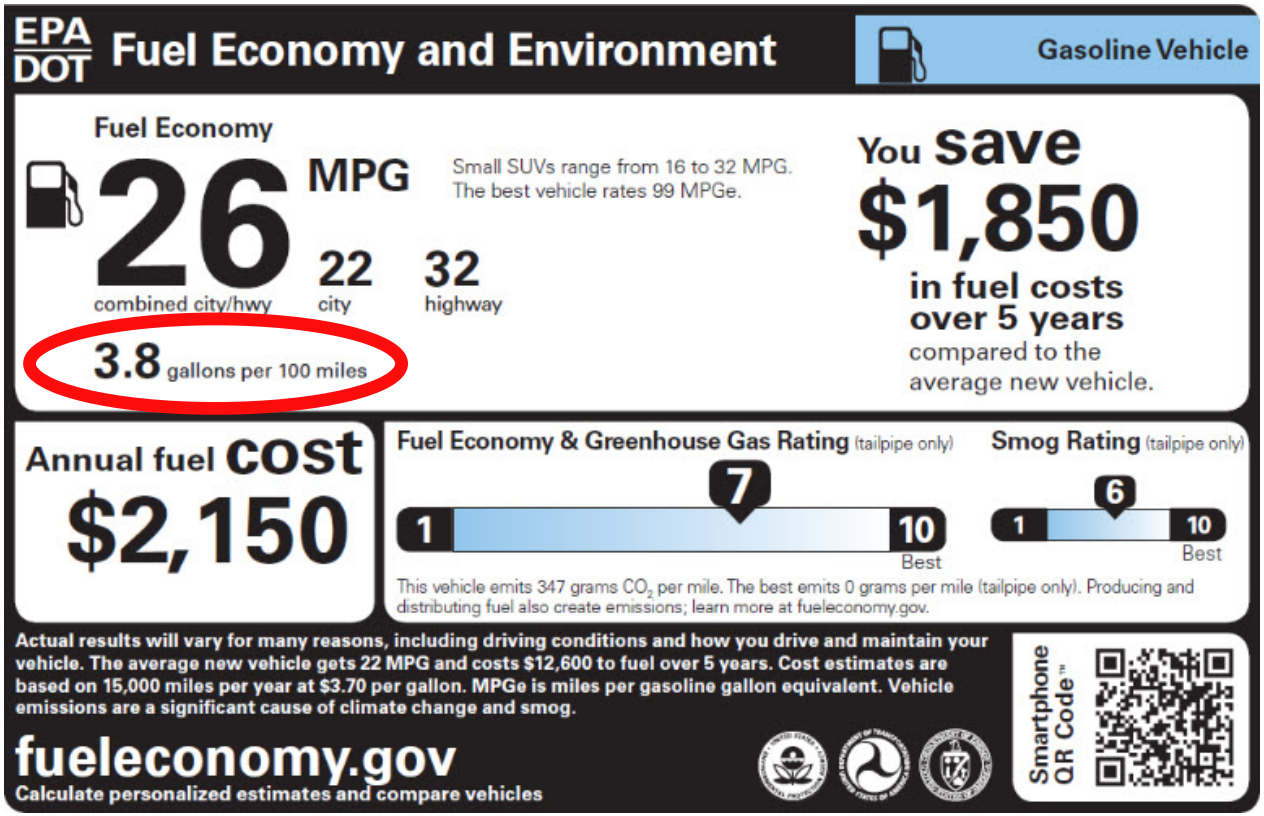
Prospect Theory

Frame of Reference



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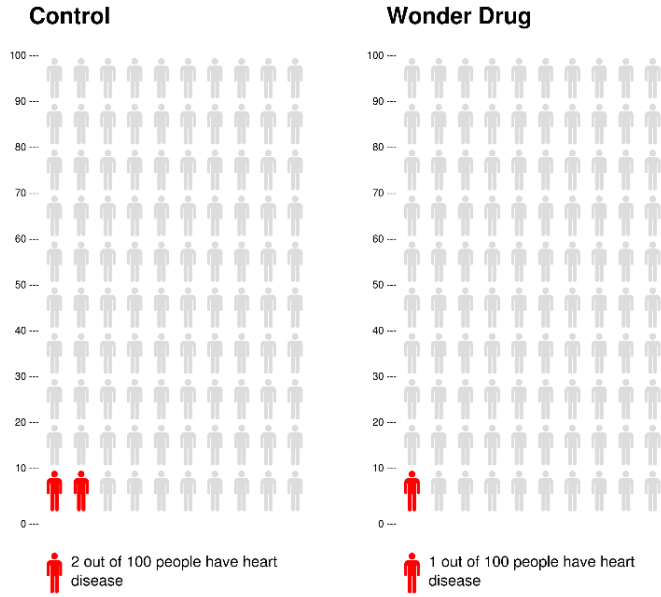
Prospect Theory
Absolute vs. Relative Frames

- New wonder drug cuts risk of heart disease in half!
- New wonder drug reduces risk of heart disease from 2% to 1%



http://clinician.iconarray.com/

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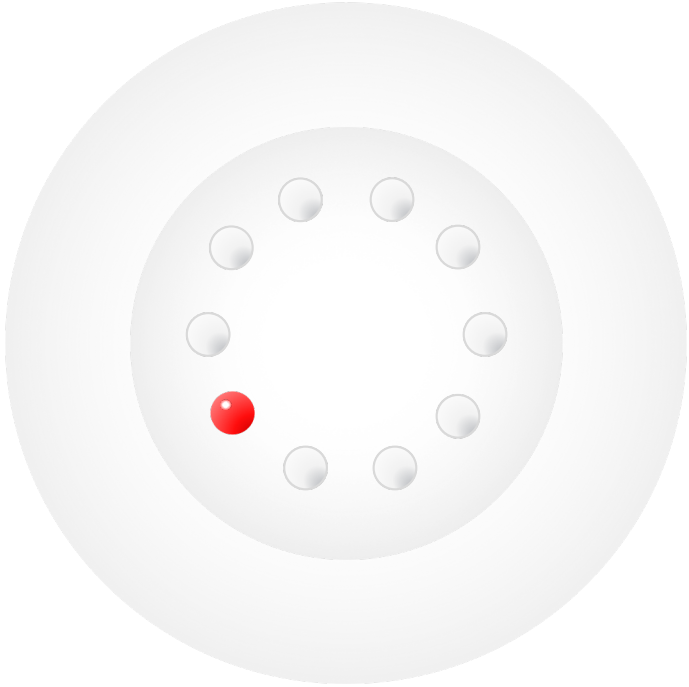
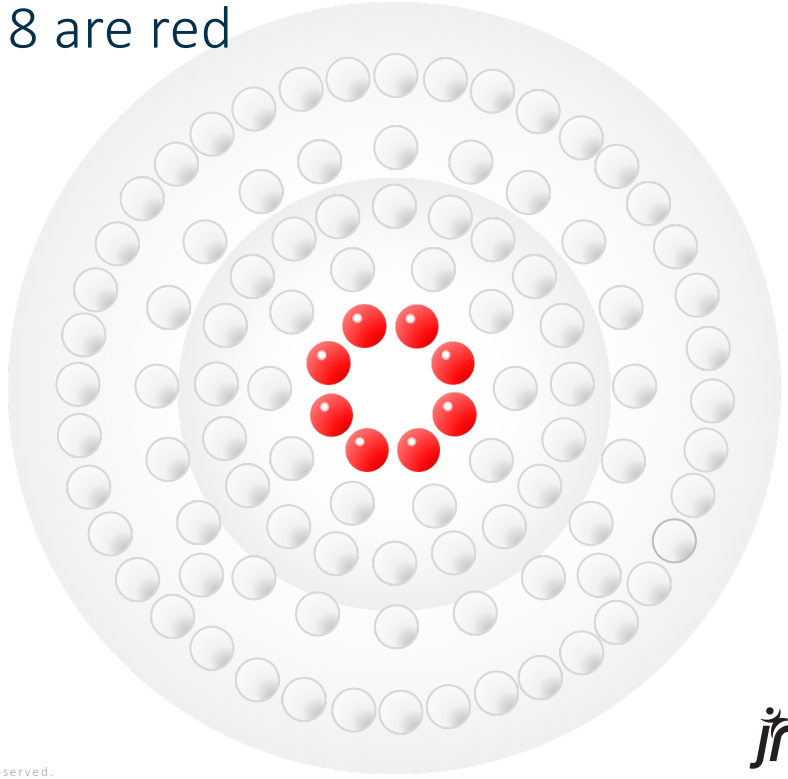
Prospect Theory
Denominator Neglect

- Urn A contains 10 marbles, of which 1 is red
- Urn B contains 100 marbles, of which 8 are red





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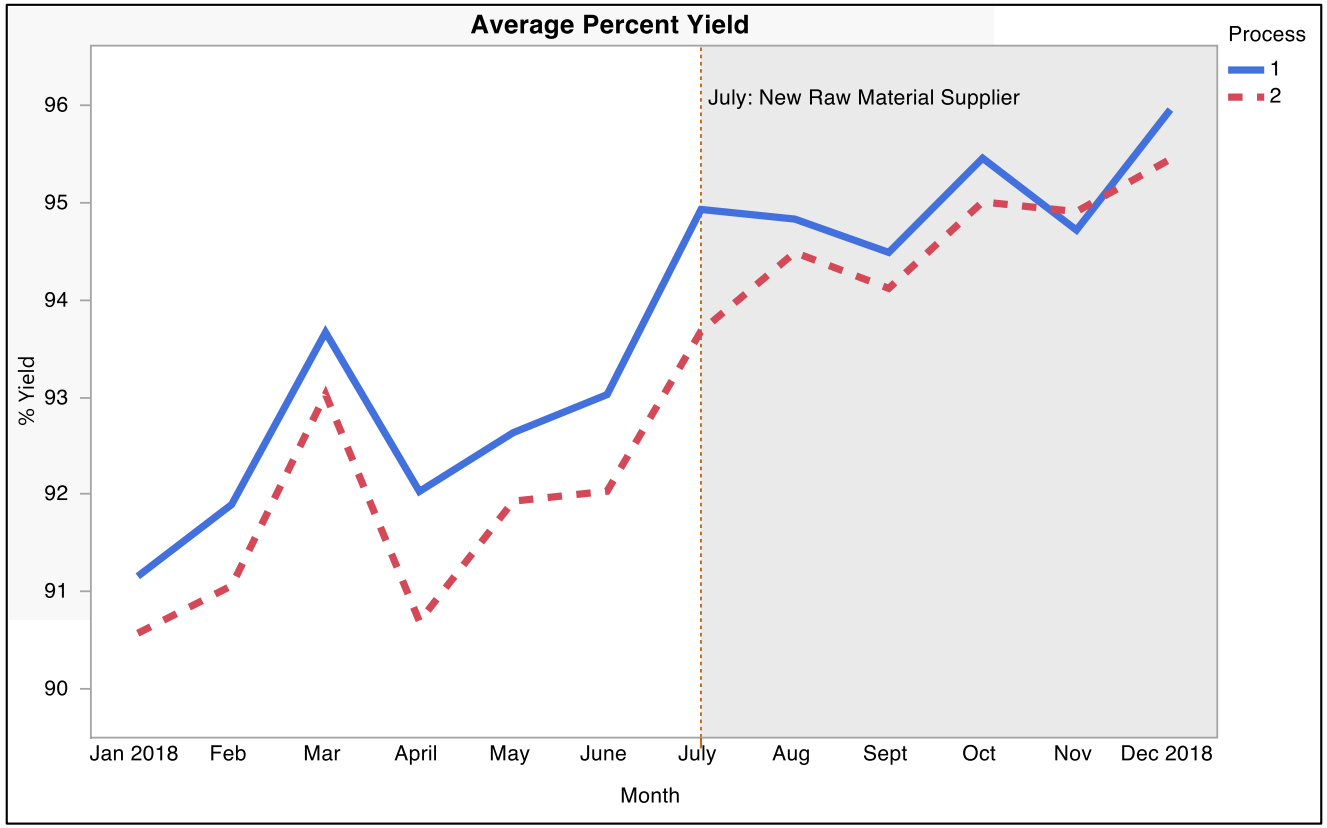
Applications

Original



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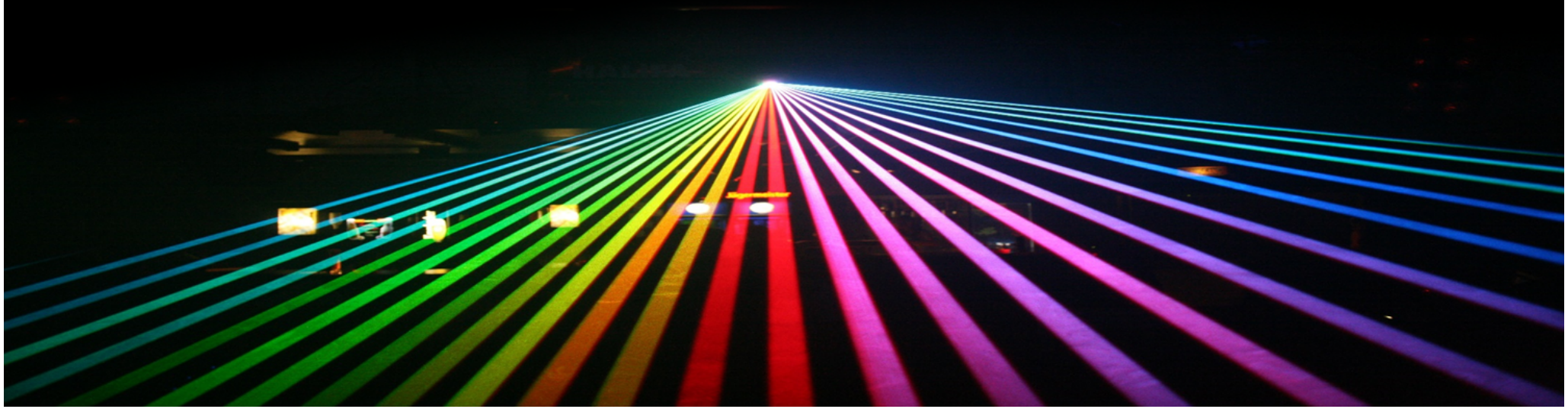
Applications
Frames

If we can choose only one process to improve, which should it be?

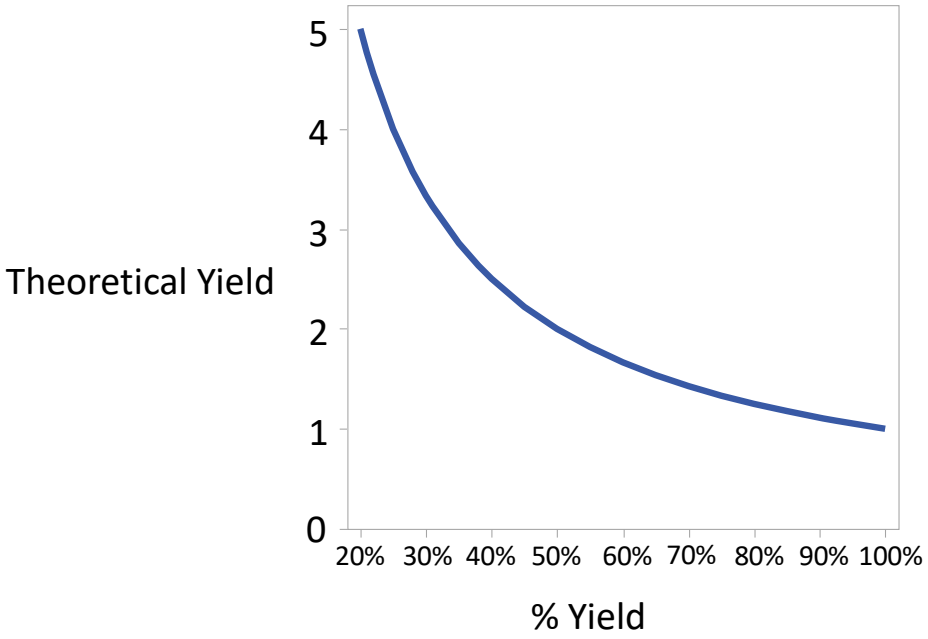
- Yield of Process 1 can be improved from 92.402% to 95.066% (difference of 2.664%)
- Yield of Process 2 can be improved from 90.553% to 93.195% (difference of 2.642%)



Applications

Frames

$$\%Yield = \frac{Actual\ Yield}{Theoretical\ Yield} \times 100$$



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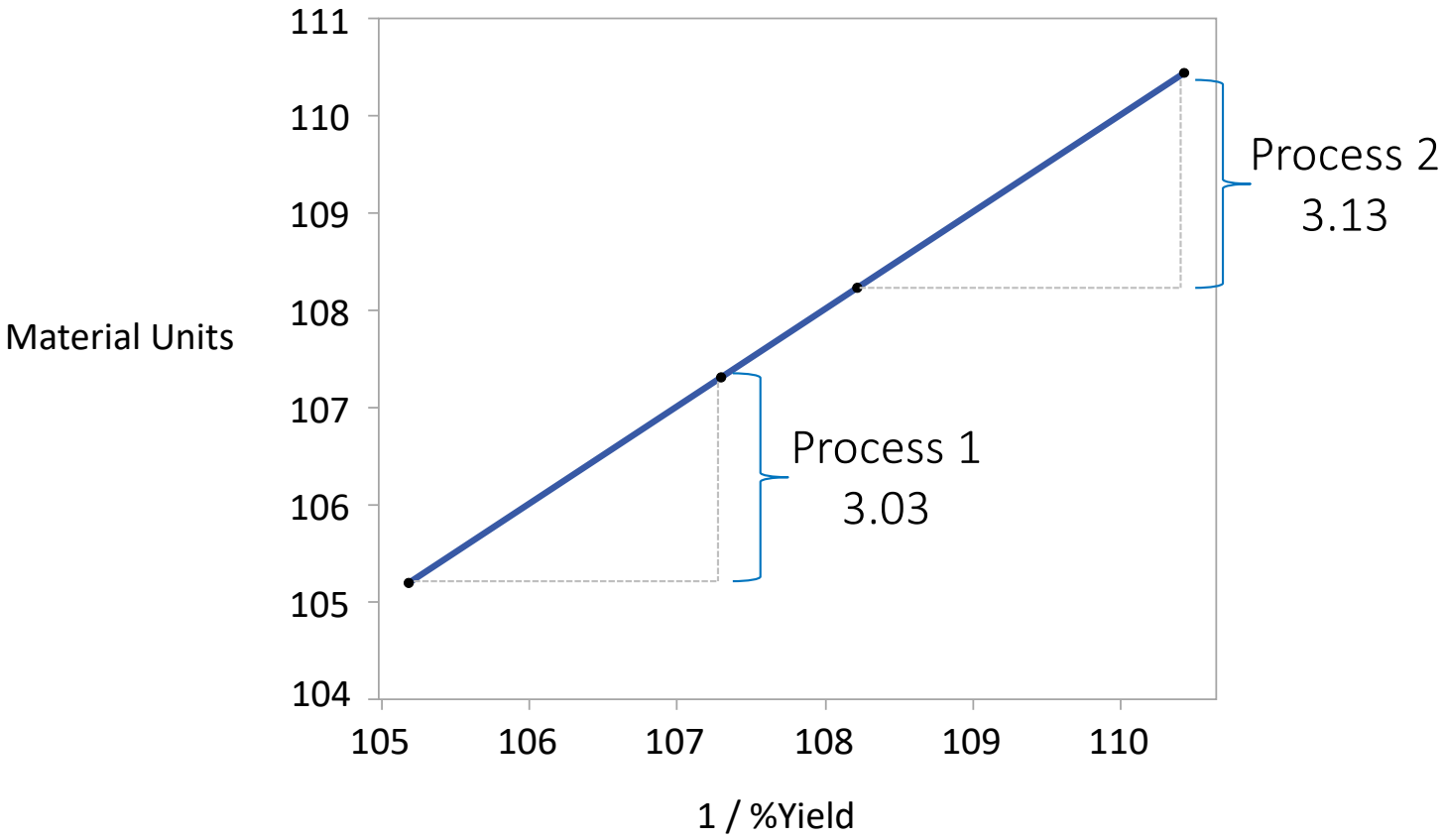
Applications

Frames



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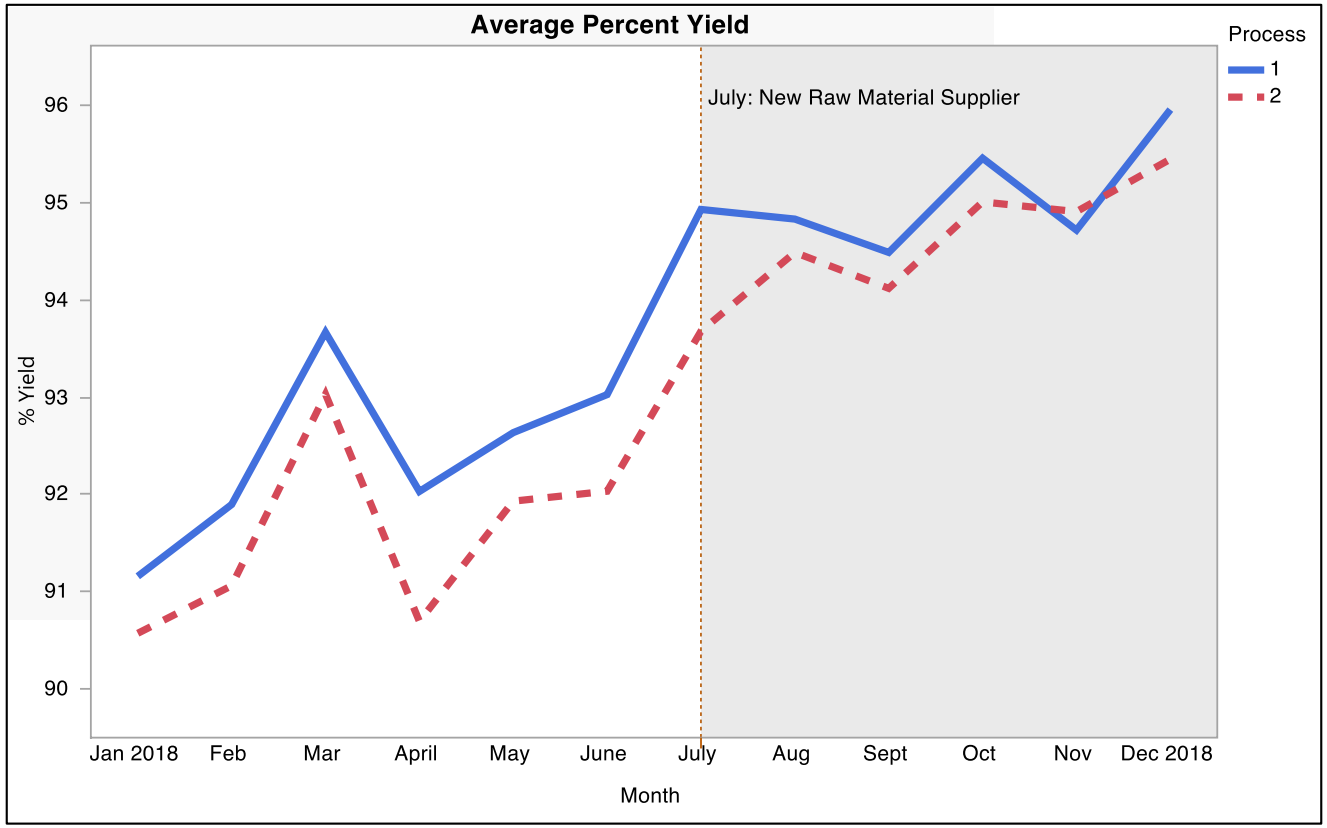
Applications

Original



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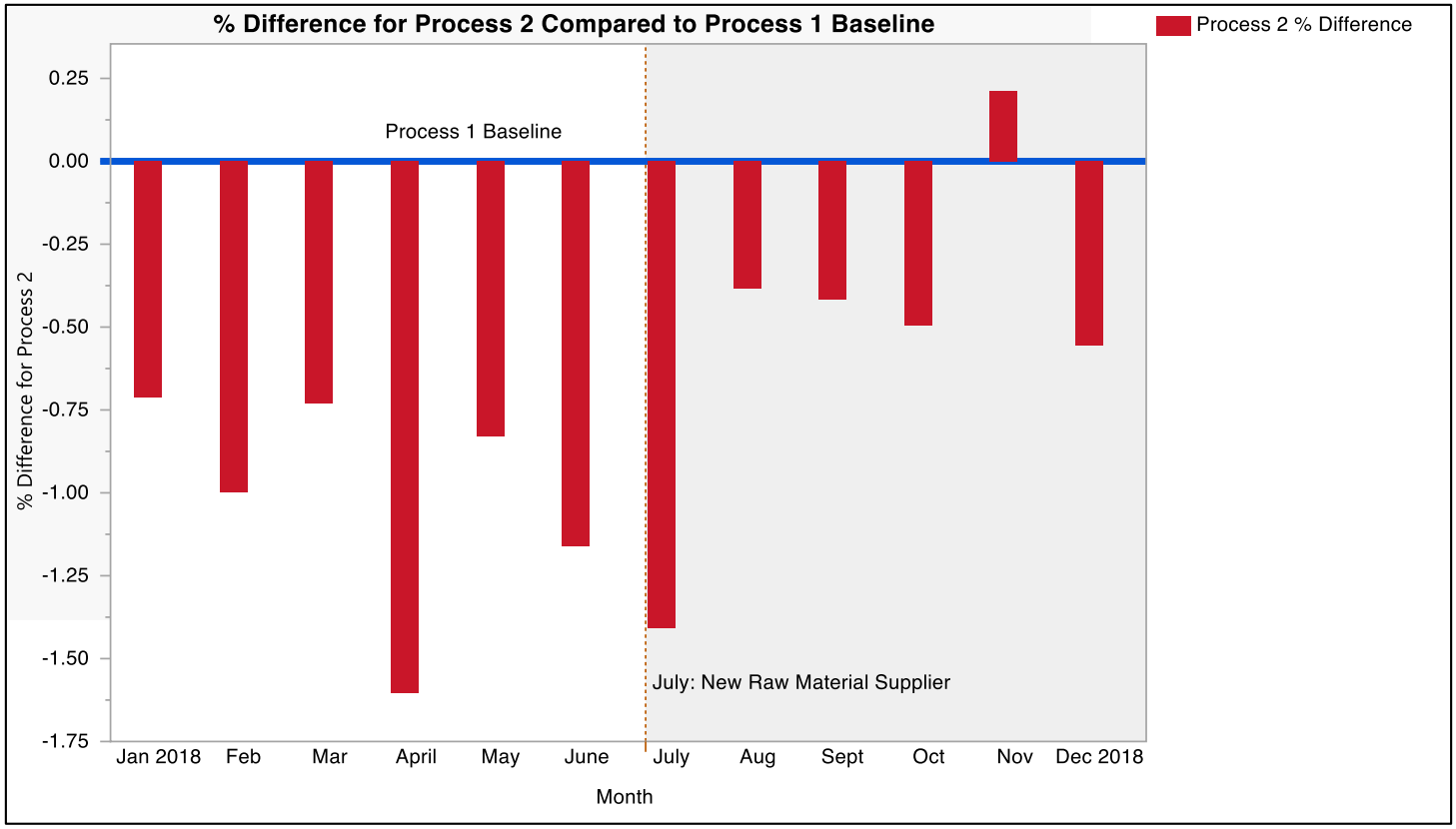
Applications

Question: Should we use Process 2?



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Applications

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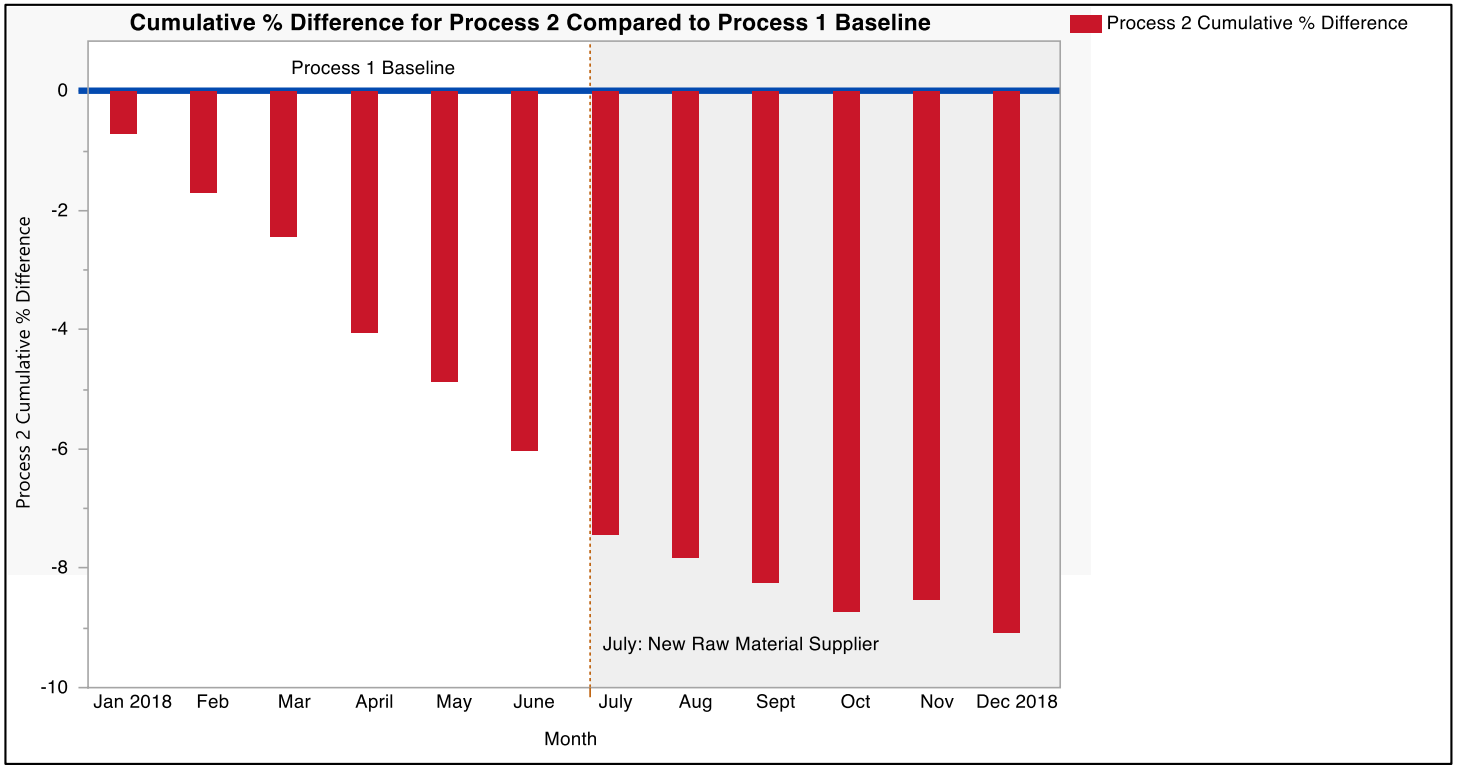

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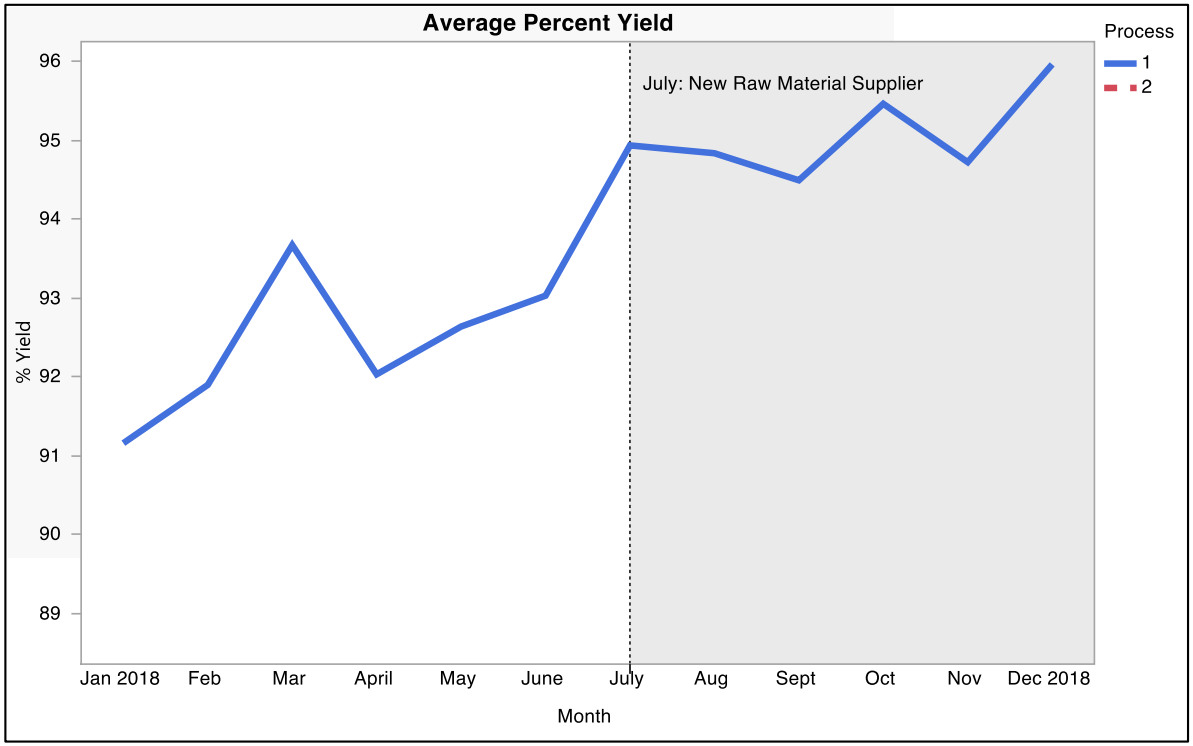
Applications

Question: Should we use the old supplier?



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Applications

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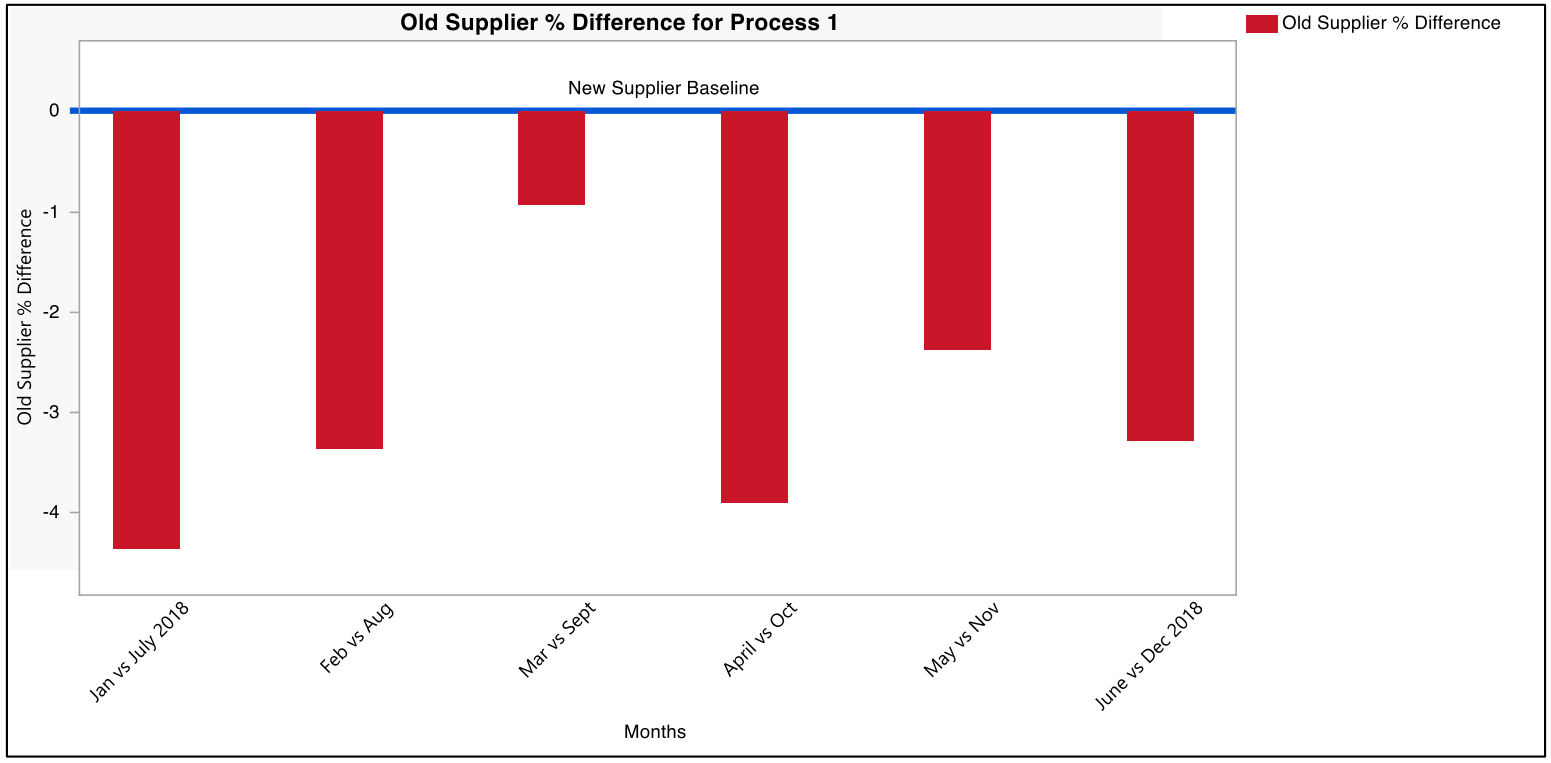

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Applications

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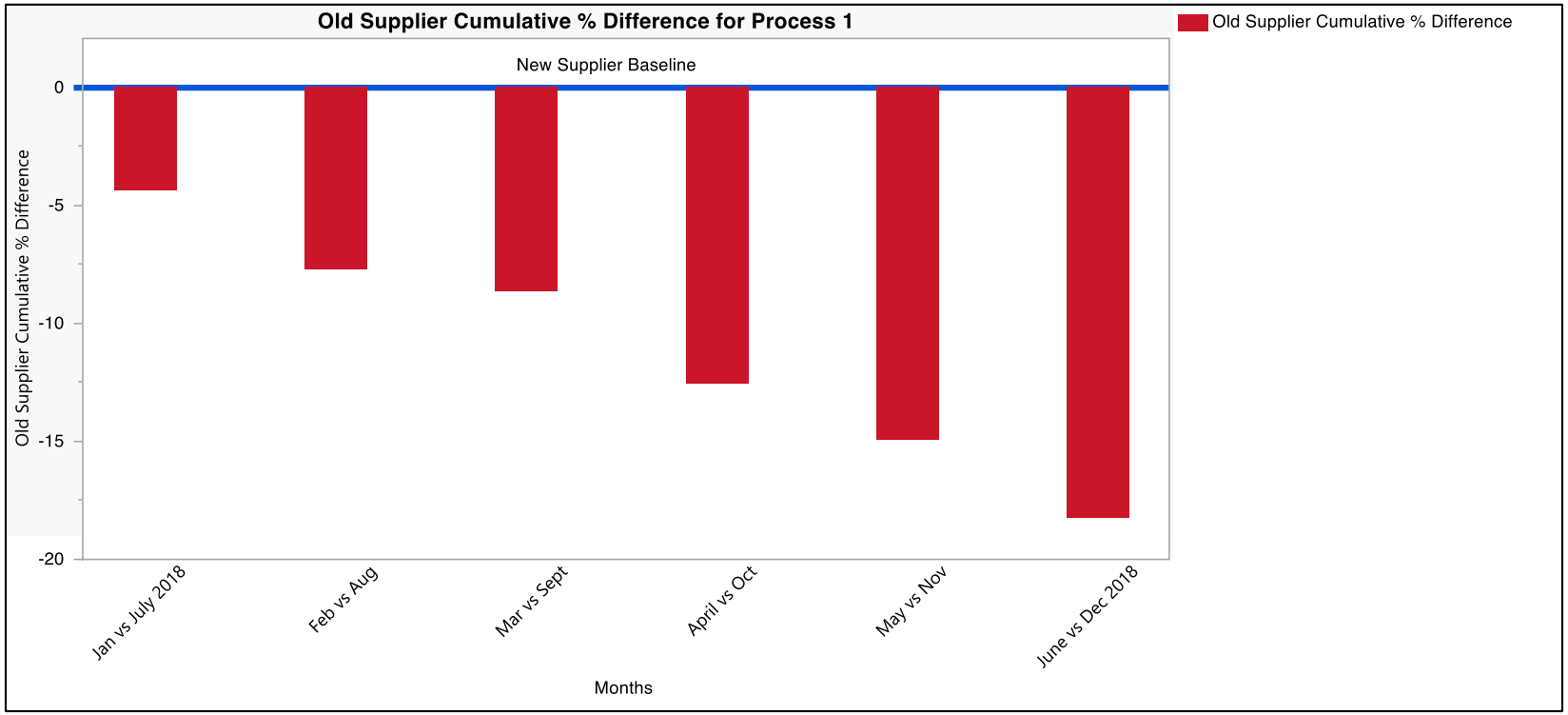

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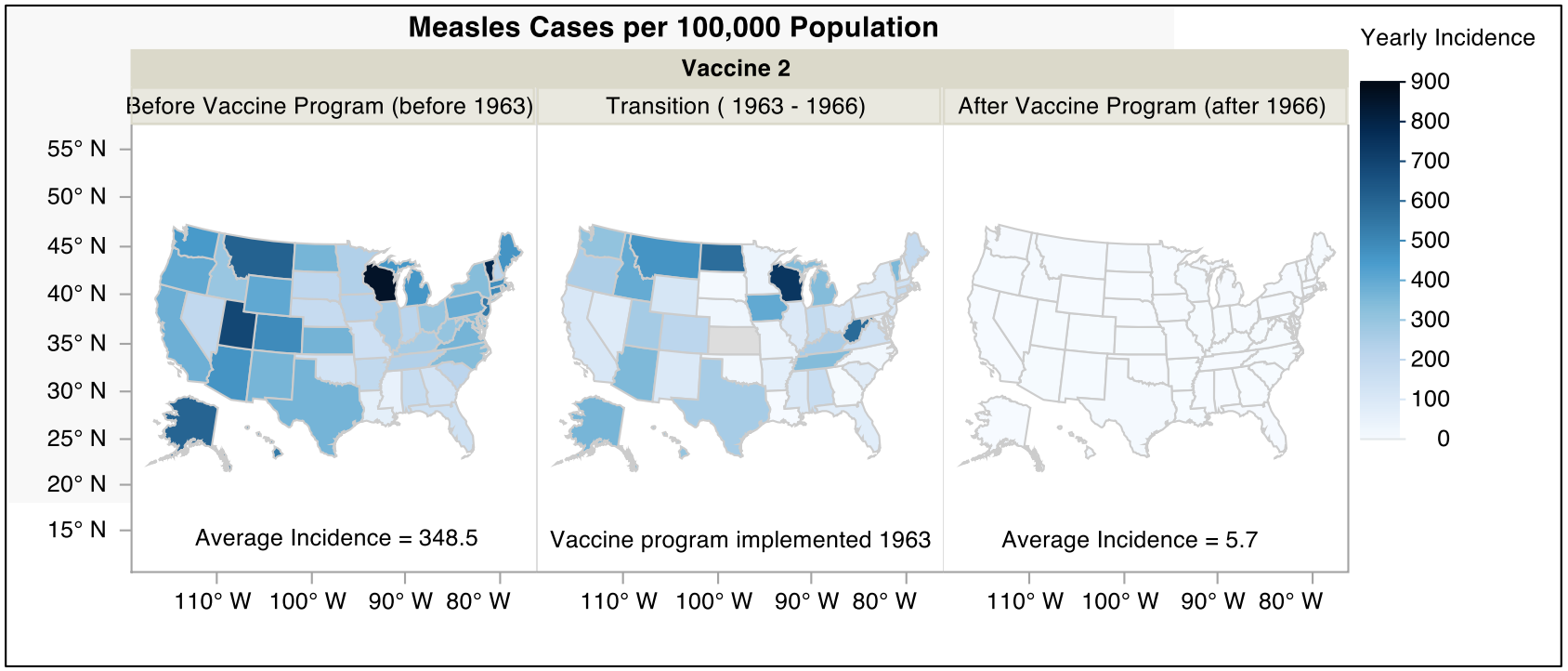
Applications

Original



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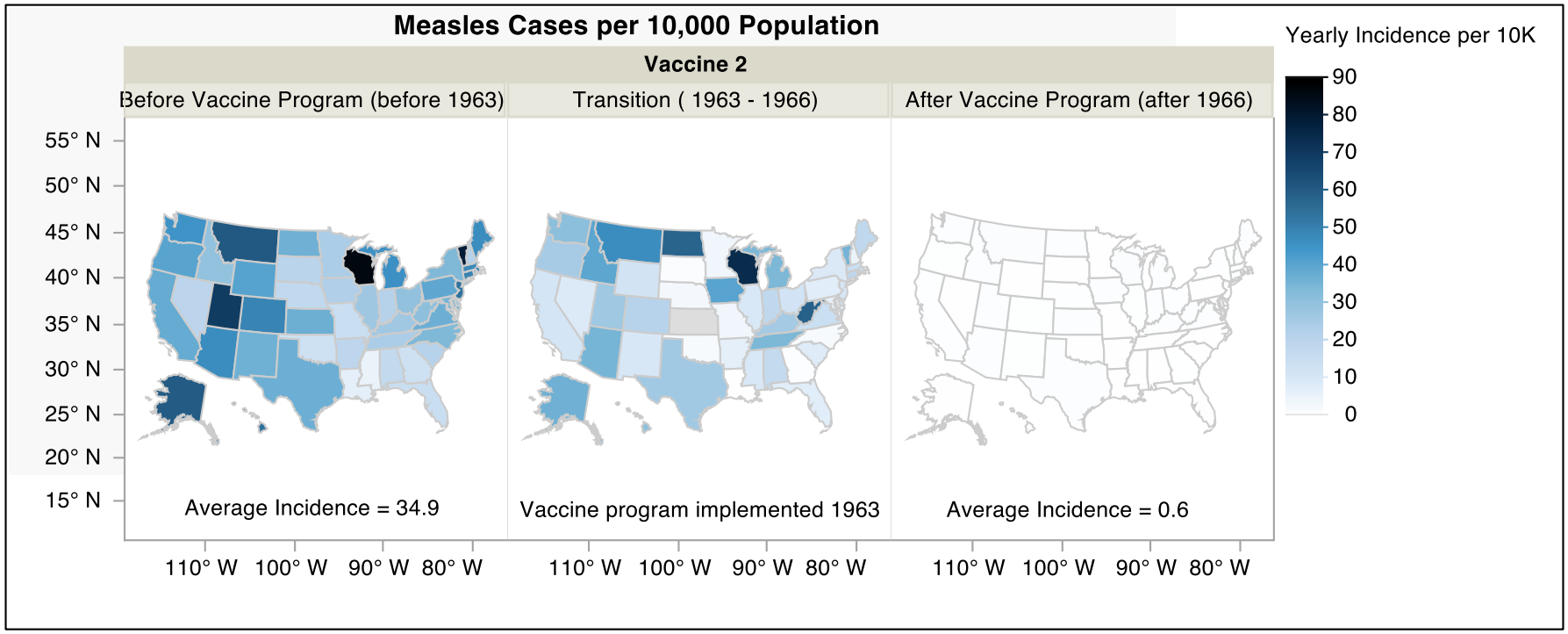
Applications

Denominator Neglect



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Prospect Theory
Loss Aversion

Capuchin monkeys exhibit loss aversion

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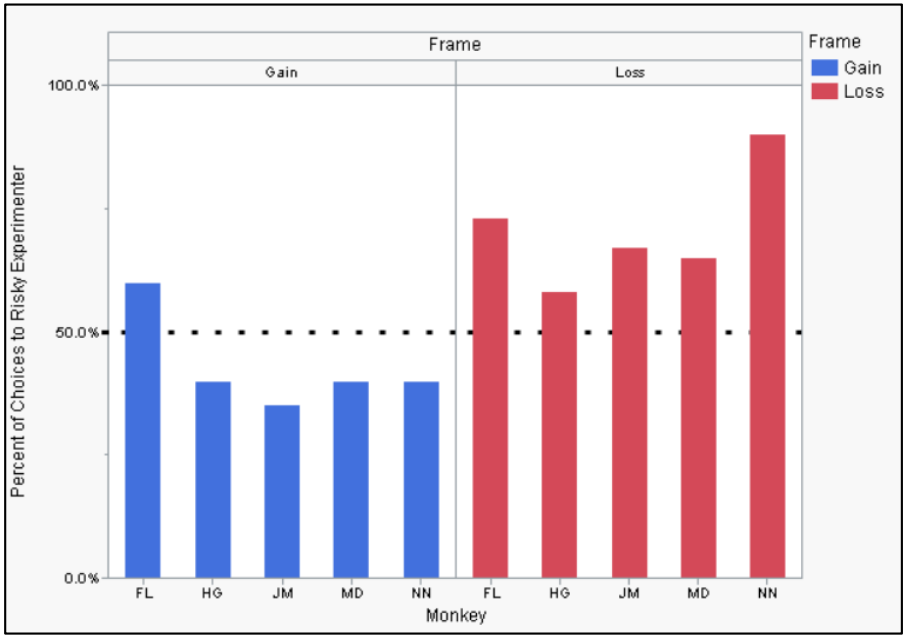

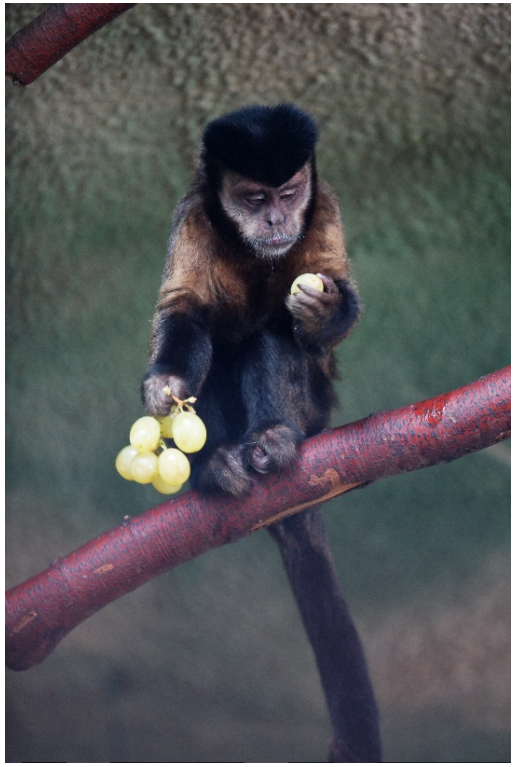

V. R. Lakshminarayanan, M. K. Chen, and L. R. Santos, "The evolution of decision-making under risk: Framing effects in monkey risk preferences," *J. Exp. Soc. Psychol.*, vol. 47, no. 3, pp. 689–693, May 2011.

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Summary

How Perceptions Influence Statistics-Based Decisions

- Value changes have diminishing utility as magnitudes increase
- Gains and losses assessed relative to status quo or expectations
- Losses are more aversive than gains are attractive
- Tendency to over weight small probabilities and under weight large probabilities

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