Using Sequential CUSUM Permutation Tests to Identify Process Shifts

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Outline

- Definitions
  - Detection methods
  - CUSUM estimator
- Sequential Permutation Method
- Simulation Results
- Simulated Case Study – JMP Demo
- Conclusions

Methods of Detection

- Visual (Simple but Subjective)
  - Raw data
  - CUSUM
  - EWMA
- Analytical (Complicated but Objective)
  - Change-Point estimators; i.e. CUSUM, EWMA
  - Mathematical Modeling; i.e. MLE, SSE
  - Permutation Tests (Proposed Method)

Types of Variation

- Common Causes – natural (random) variations that are part of a stable process
  - ✓Machine vibration
  - ✓Temperature, humidity, electrical current fluctuations
  - ✓Slight variation in raw materials

- Special Causes – unnatural (non-random) variations that are not part of a stable process
  - ✓Batch of defective raw material
  - ✓Faulty set-up
  - ✓Human error
  - ✓Incorrect recipe

Cumulative Sum Control Chart

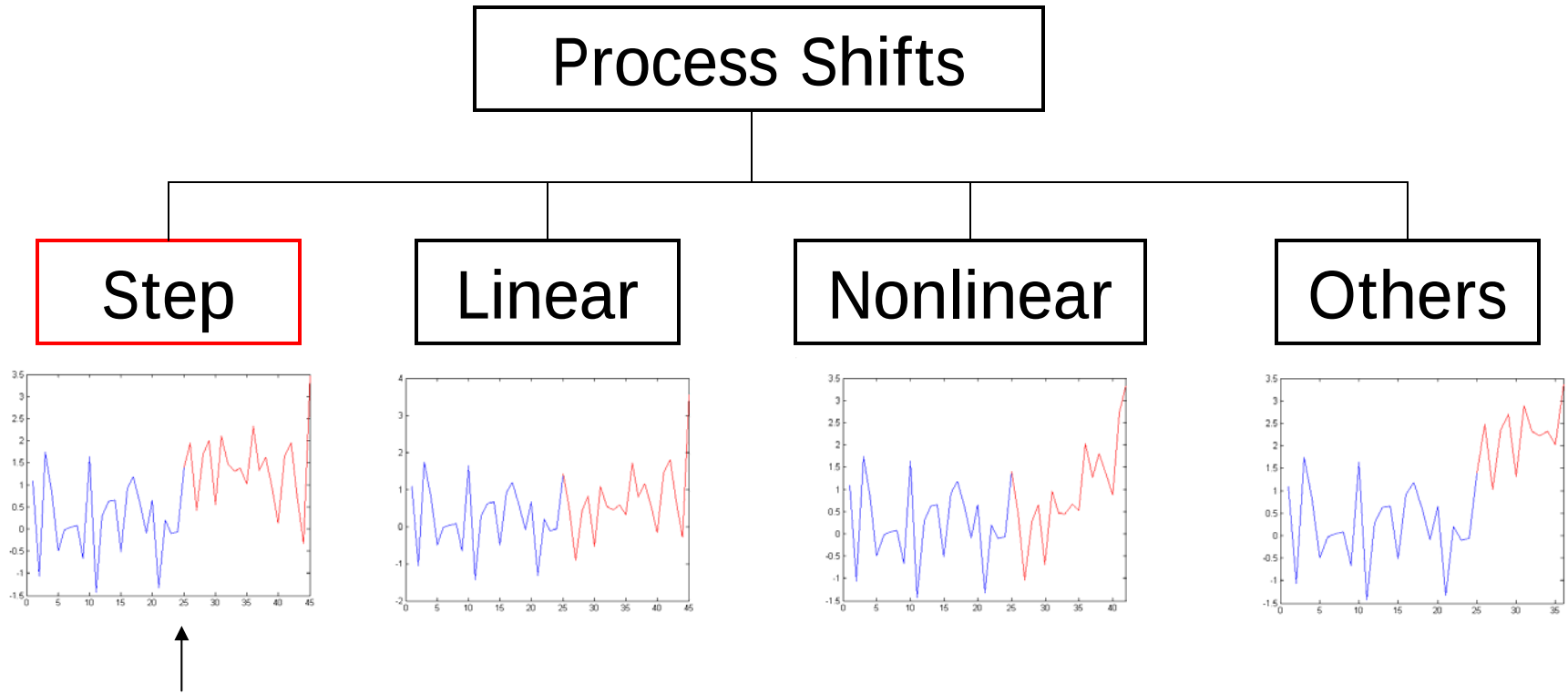
- CUSUM: cumulative sum of deviations from average

$$C_t^+ = \max\left[0, \bar{y}_t - (\mu_0 + K) + C_{t-1}^+\right]$$

$$C_t^\pm = \text{accumulate d deviations above/belo w } \mu_0$$

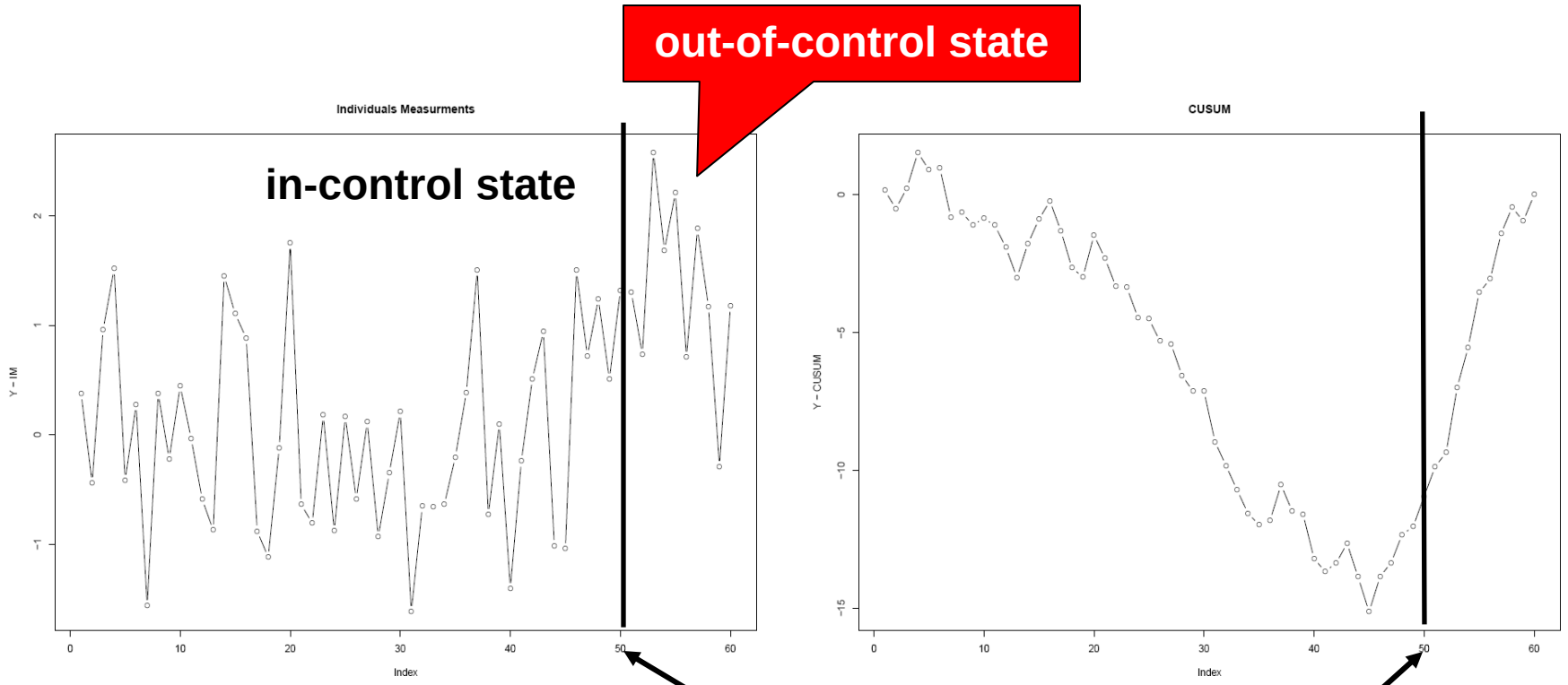
- A bit more difficult to set up
- More difficult to understand
- Very effective when subgroup size *n*=1
- Very good for detecting small shifts
- Change-point detection capability
- Less sensitive to autocorrelation

Types of Process Shifts



Our Example Today

Process Model



Change point,
"Unknown"

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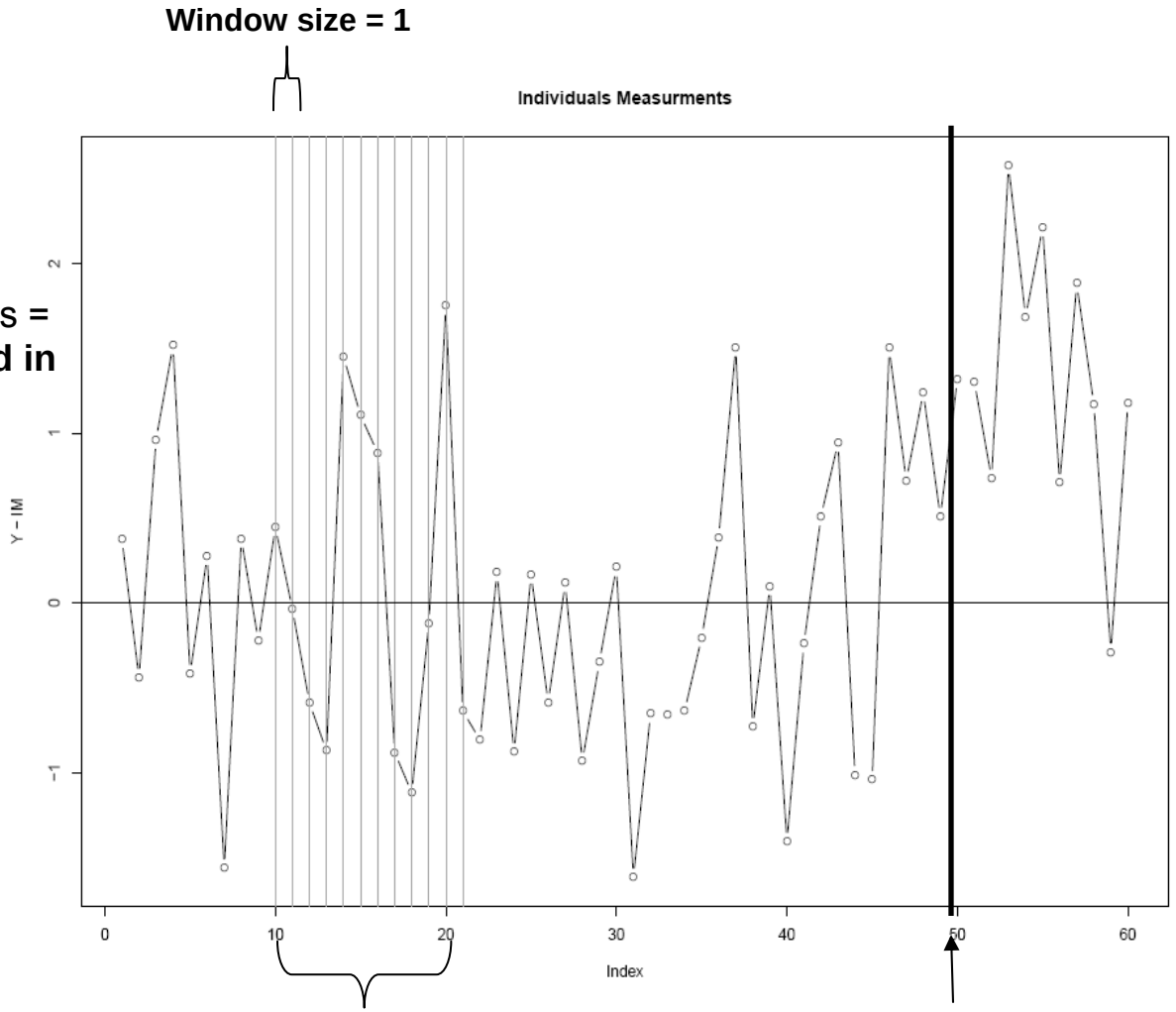
Sequential Permutation Test

- Sequential Test
  - In order to determine where the change point may have occurred choose a region within the time series where you wish to determine if a shift in the process occurred
  - Between adjacent points (window), within this region, determine the value difference between your CUSUM statistic
  - Randomly permute the CUSUM, each time calculating the value difference for the corresponding points
  - Use the percentile method to determine the proportion of times your calculated difference was greater than the calculated differences in the random permutations
    - If >95%, you can conclude that a shift may have occurred

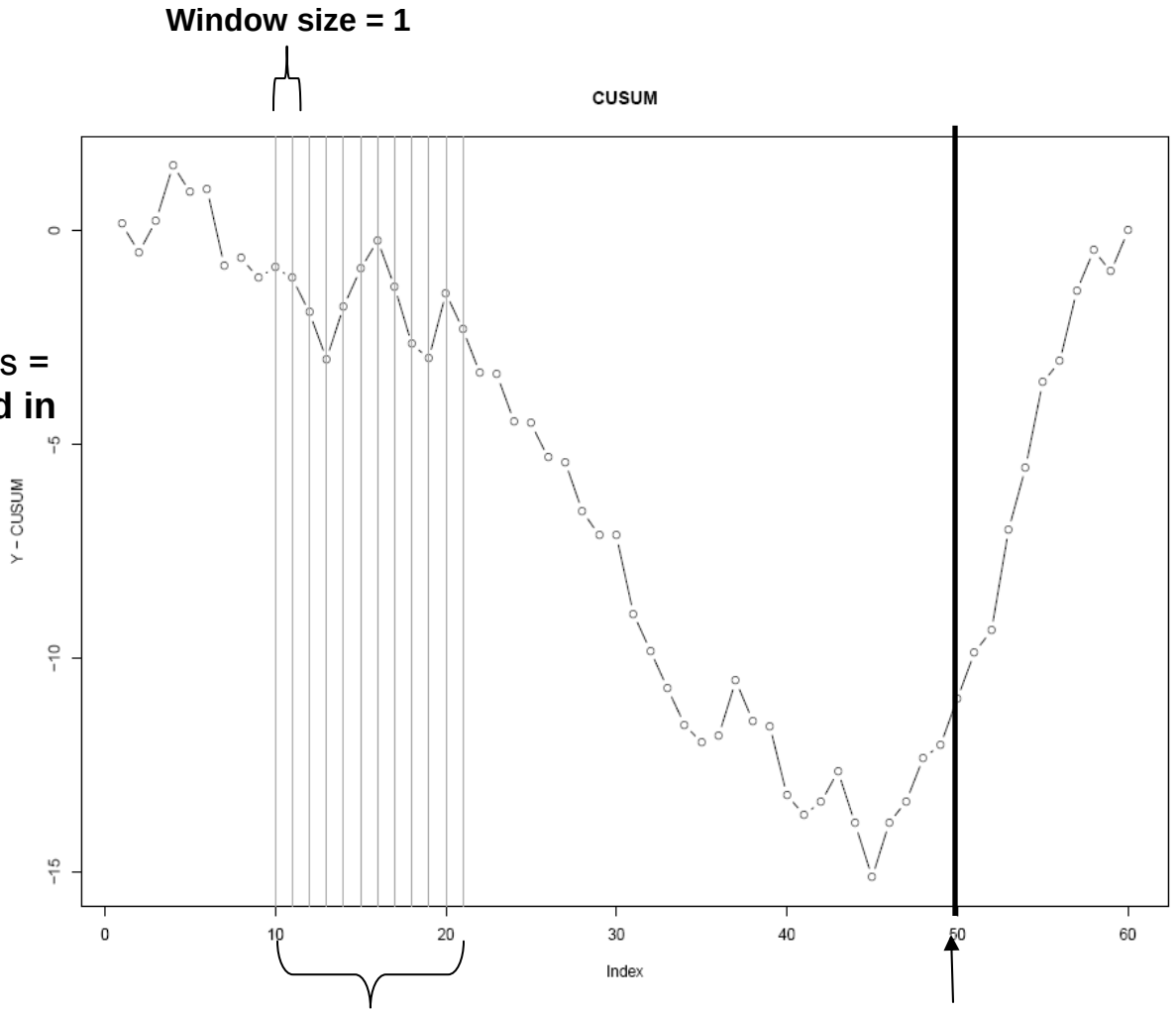
Stable Area Individuals Measurement Chart

Grey Vertical Lines = **No Shift Detected in CUSUM**



Stable Area CUSUM Chart

Grey Vertical Lines = **No Shift Detected in CUSUM**



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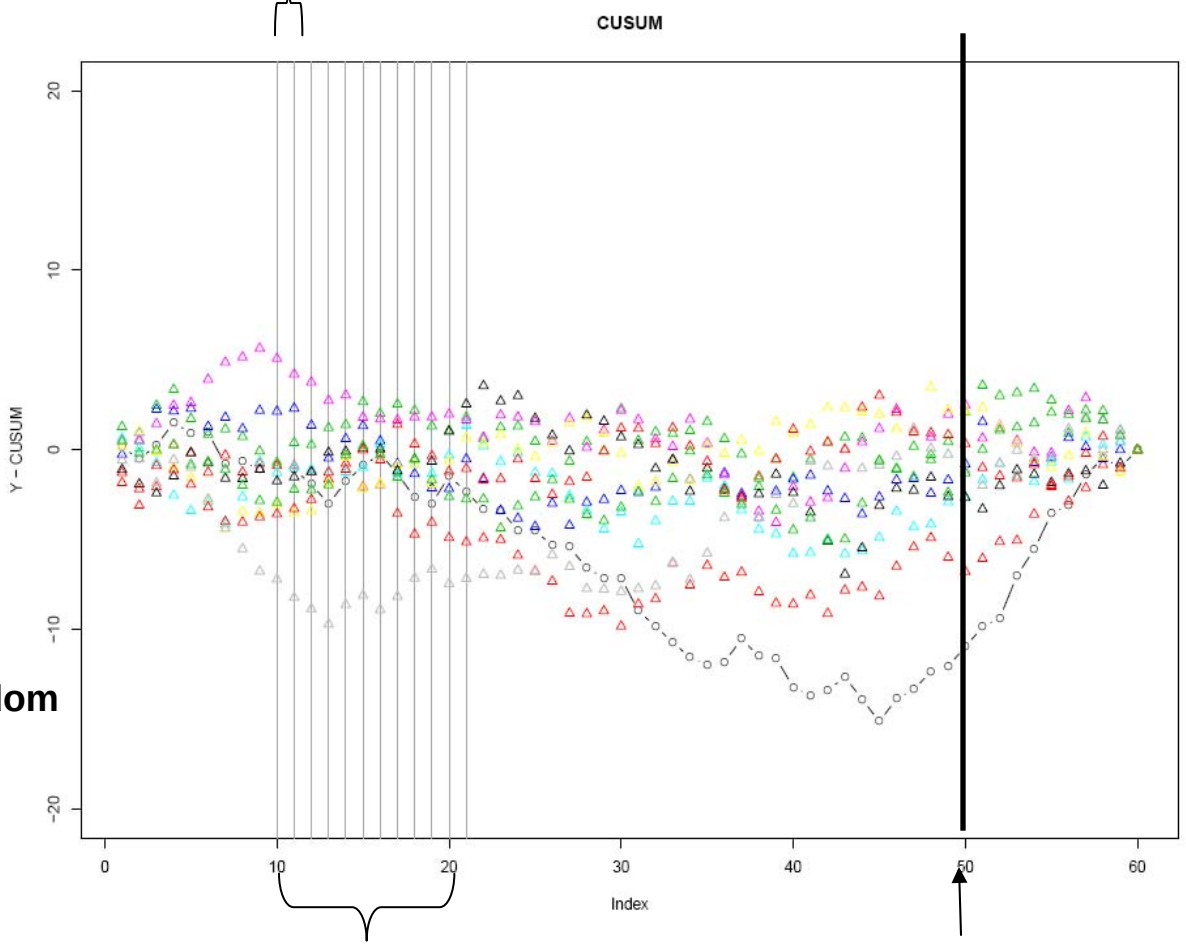
CUSUM Chart with Random Permutations

Window size = 1



Grey Vertical Lines = No Shift Detected in CUSUM

Black Circles = Actual CUSUM

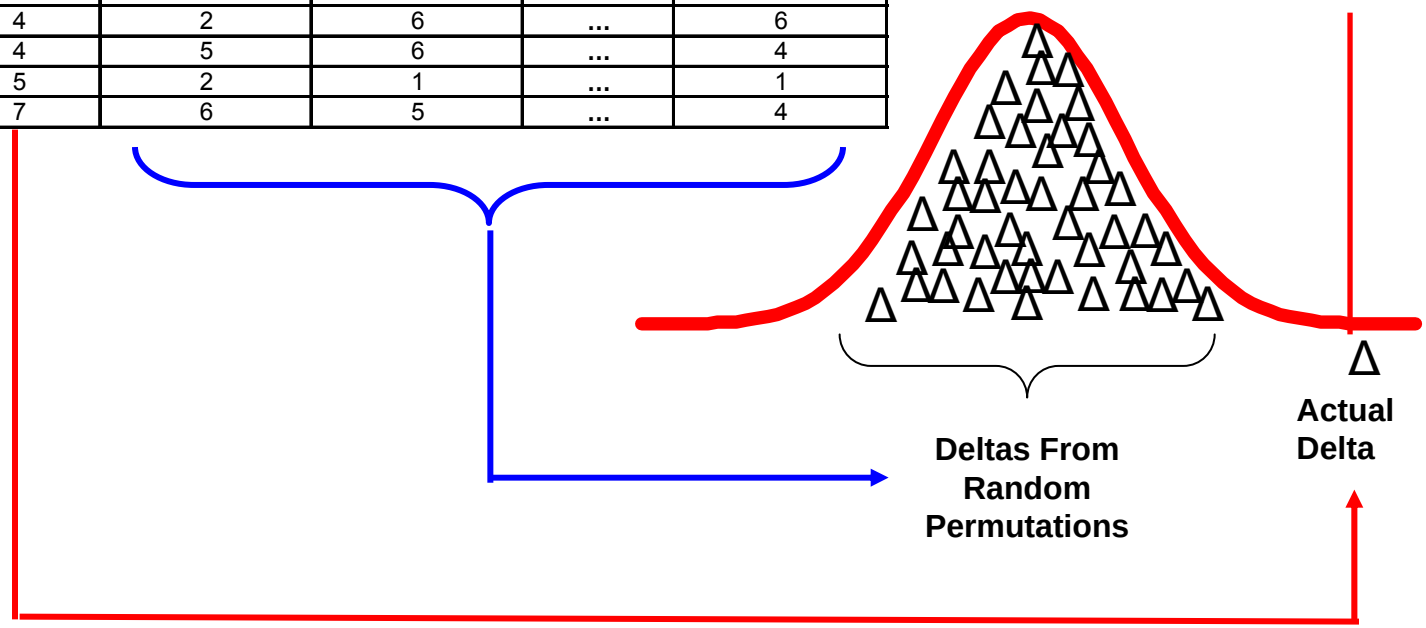
Triangles = Random Permutations

Region Explored

Shift Introduced

Distribution of Randomly Permuted and Actual Delta's Calculated from CUSUM

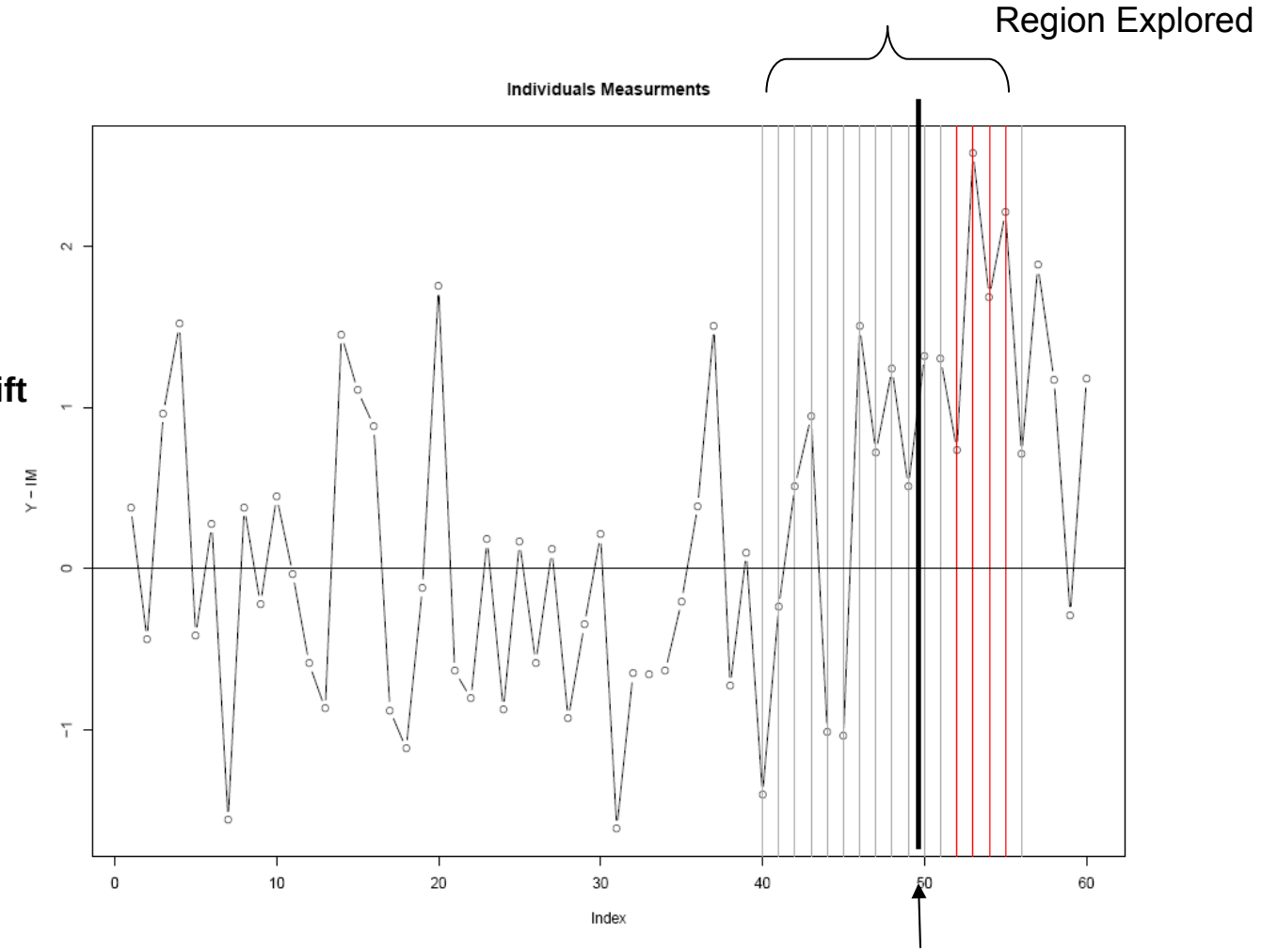
Window						
X	Y	Actual Delta	Random Delta 1	Random Delta 2	…	Random Delta n
40	45	6	1	1	…	4
41	46	1	2	5	…	2
42	47	2	3	2	…	7
43	48	1	5	3	…	0
44	49	5	5	4	…	4
45	50	2	3	0	…	6
46	**51**	**10**	**2**	**4**	**…**	**1**
47	52	4	2	6	…	6
48	53	4	5	6	…	4
49	54	5	2	1	…	1
50	55	7	6	5	…	4



Shift Area Individuals Measurement Chart

Grey Vertical Lines = No Shift Detected in CUSUM

Red Vertical Lines = Shift Detected in CUSUM



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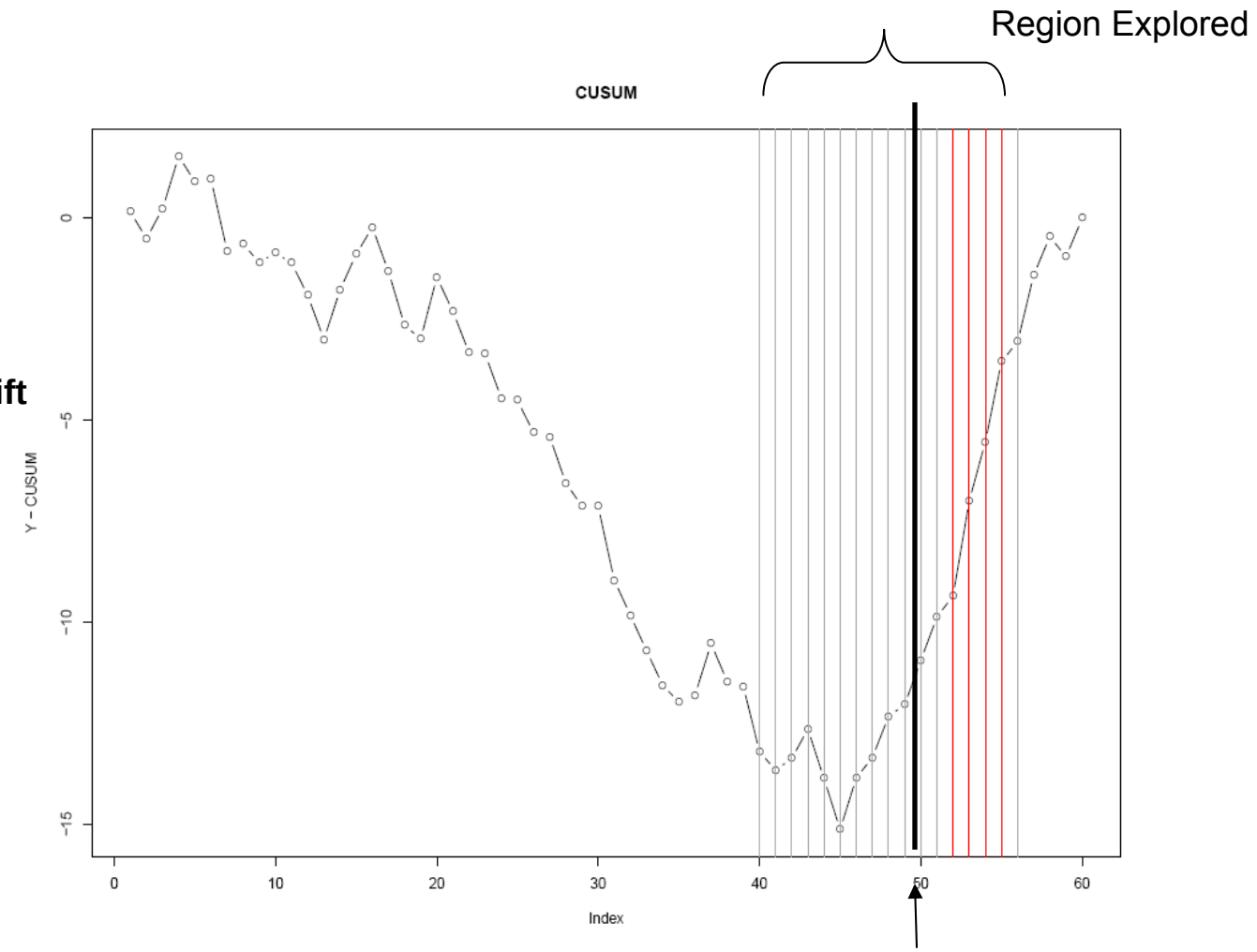
Shift Area CUSUM Chart

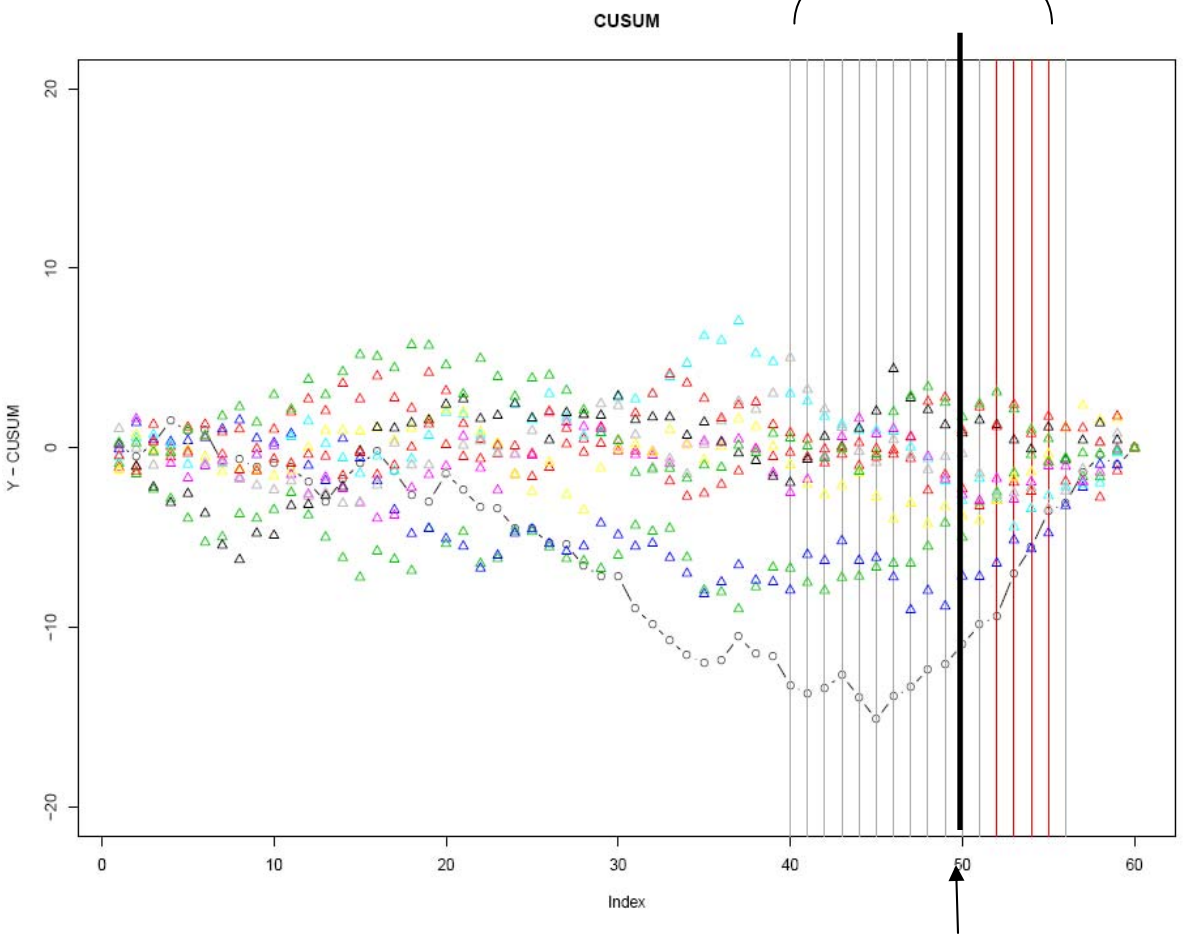
Grey Vertical Lines = No Shift Detected in CUSUM

Red Vertical Lines = Shift Detected in CUSUM



Shift Area CUSUM w/ 10 Random Permutations

Region Explored



Shift Introduced

Black Circles = Actual CUSUM

Triangles = Random Permutations

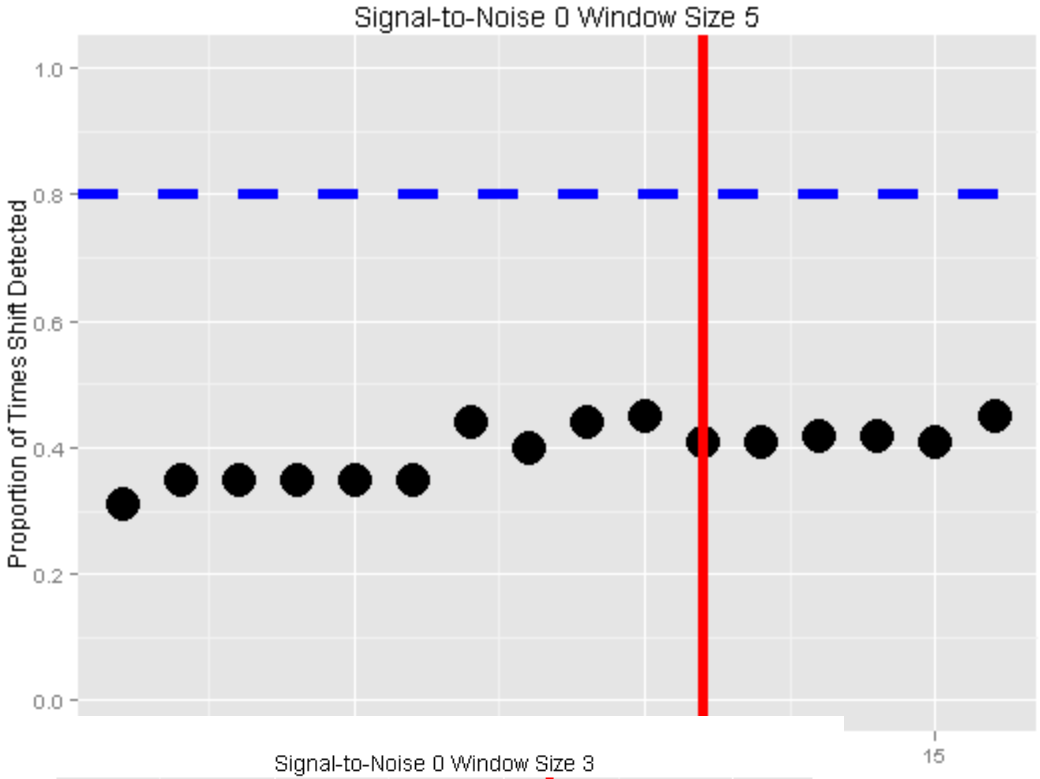
Grey Vertical Lines = No Shift Detected in CUSUM

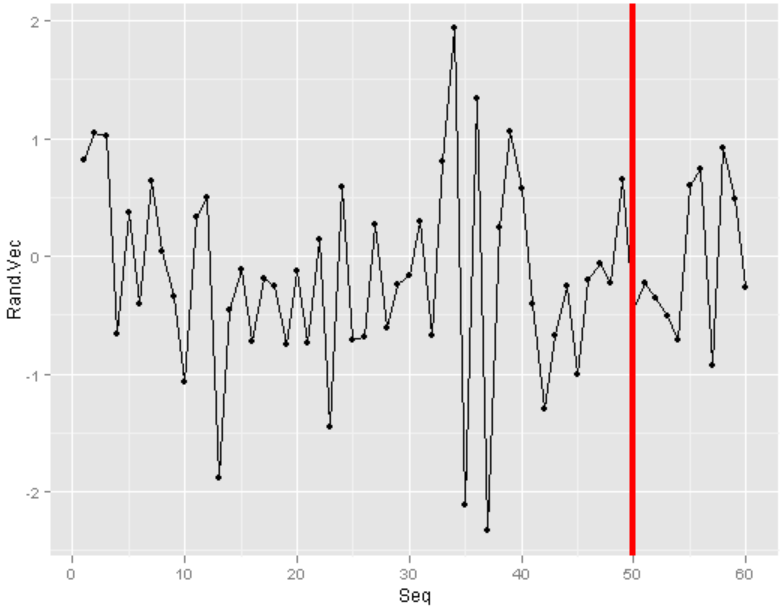
Red Vertical Lines = Shift Detected in CUSUM

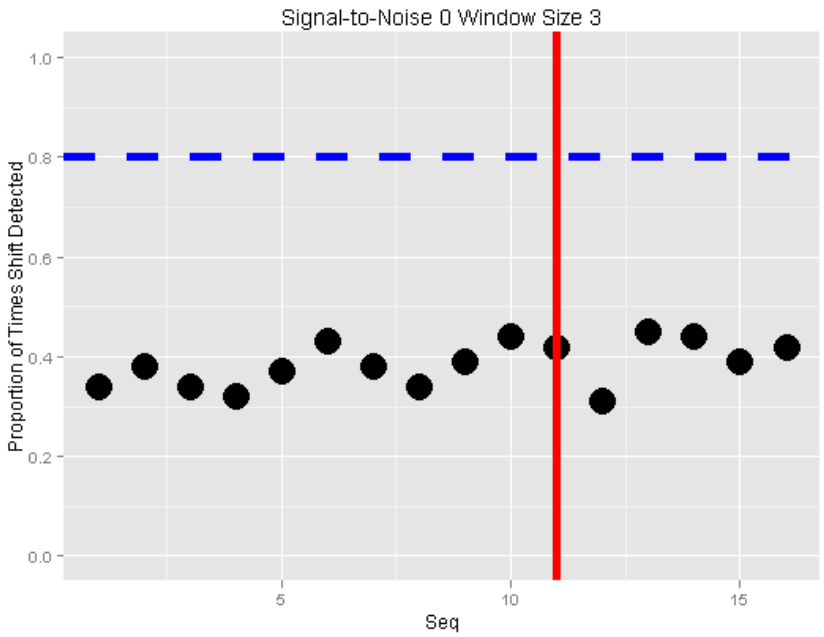
Sensitivity Analysis

- 1000 Simulation Runs
- 1000 Permutations
- Window Size: 3 and 5
- Test @ S/N: 0, 1, 1.5 and 2

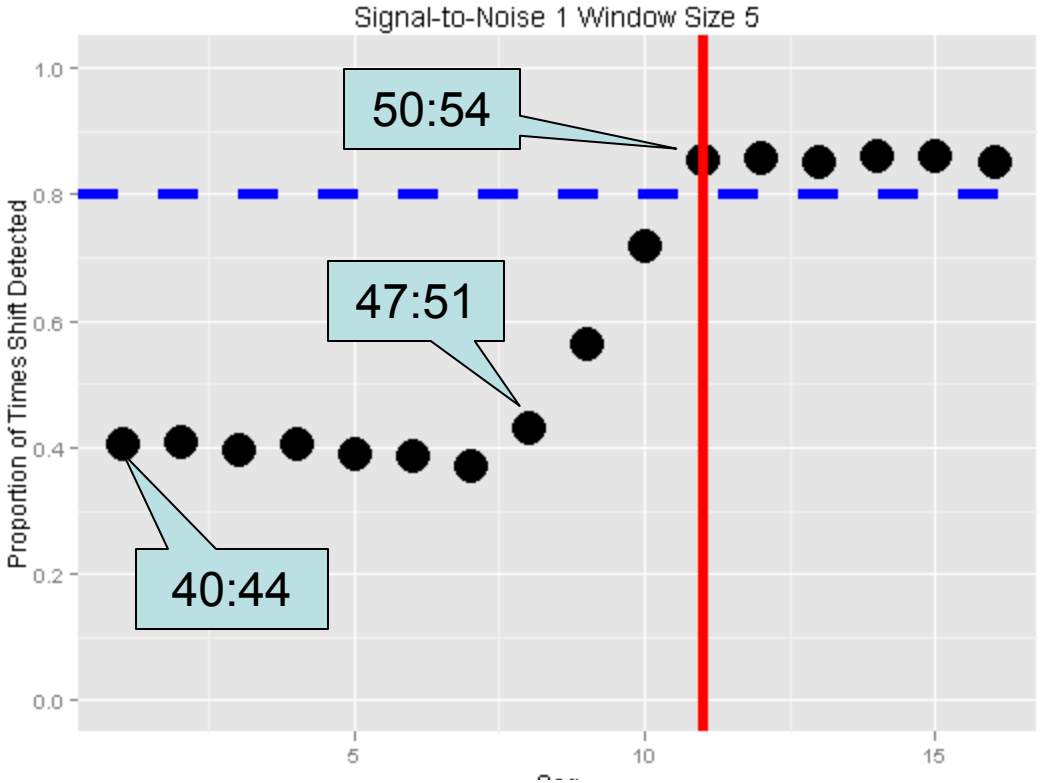
S/N=0, Wind=5

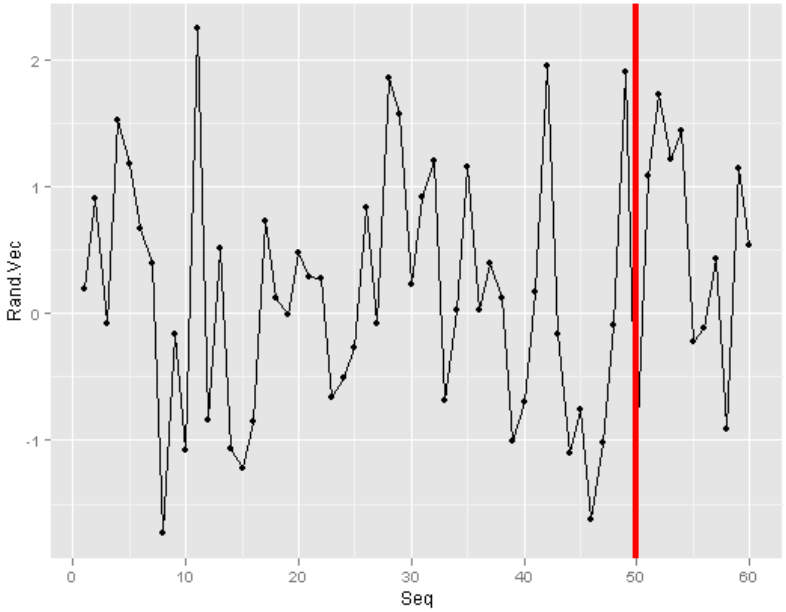


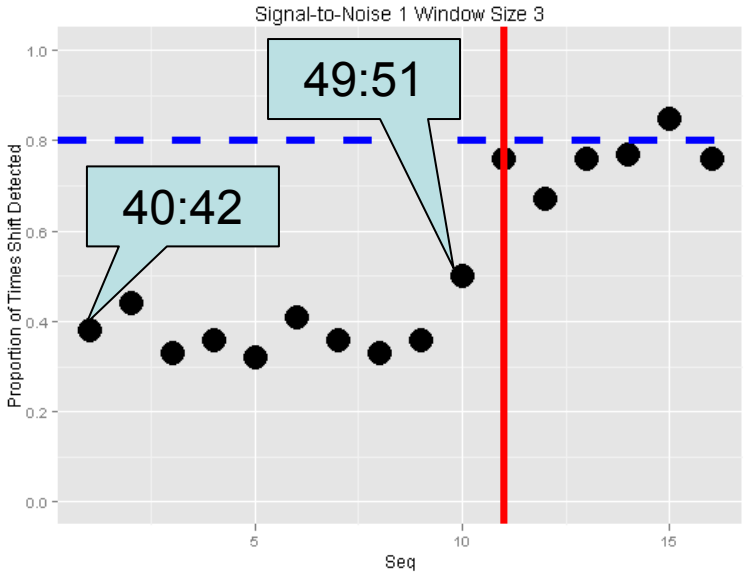




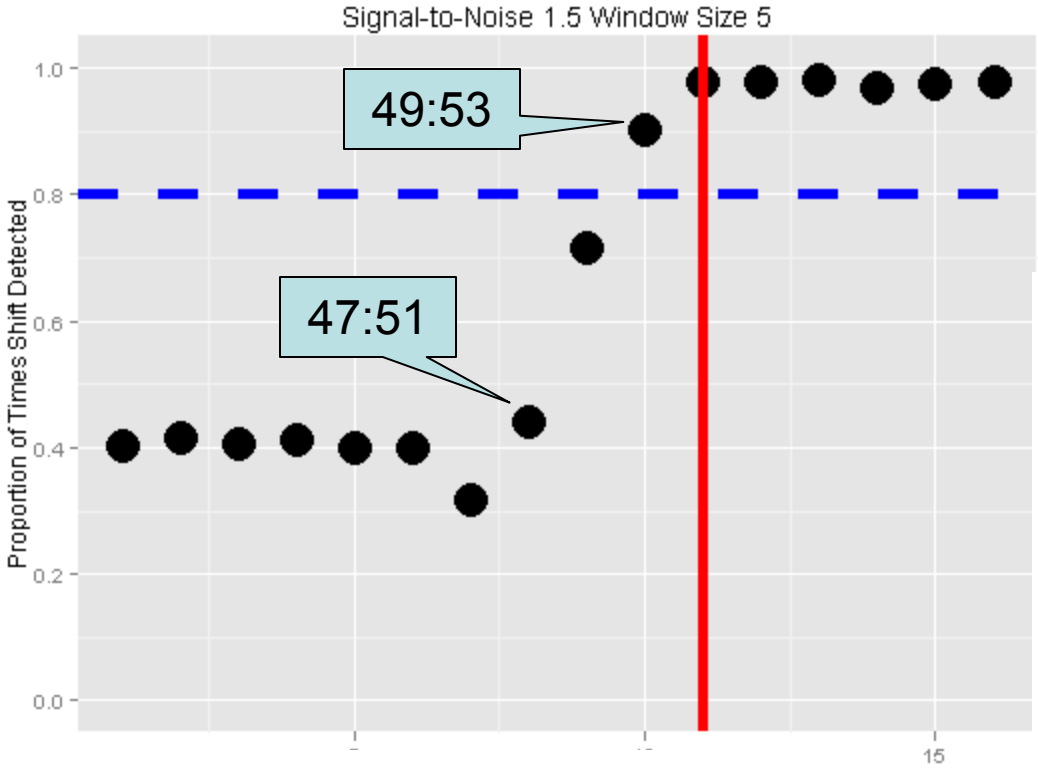
S/N=1, Wind=5

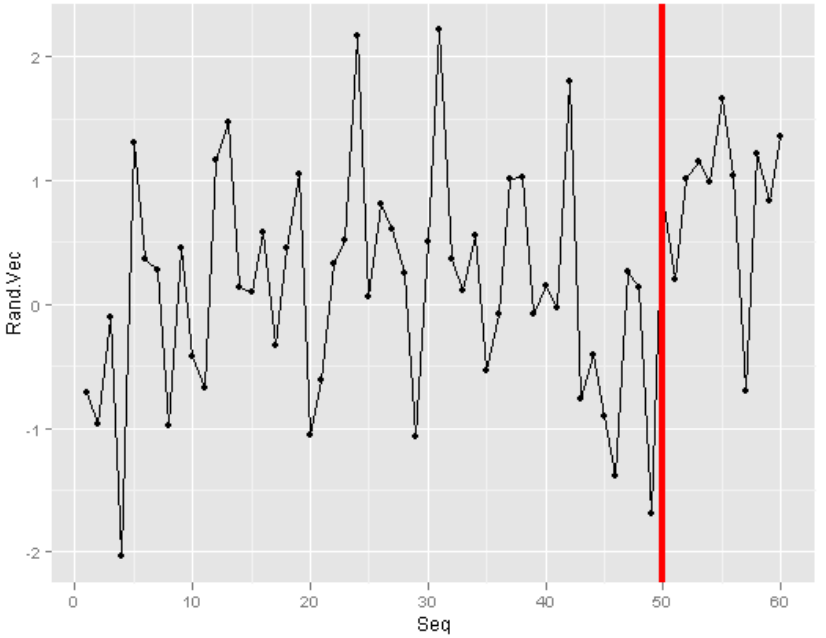


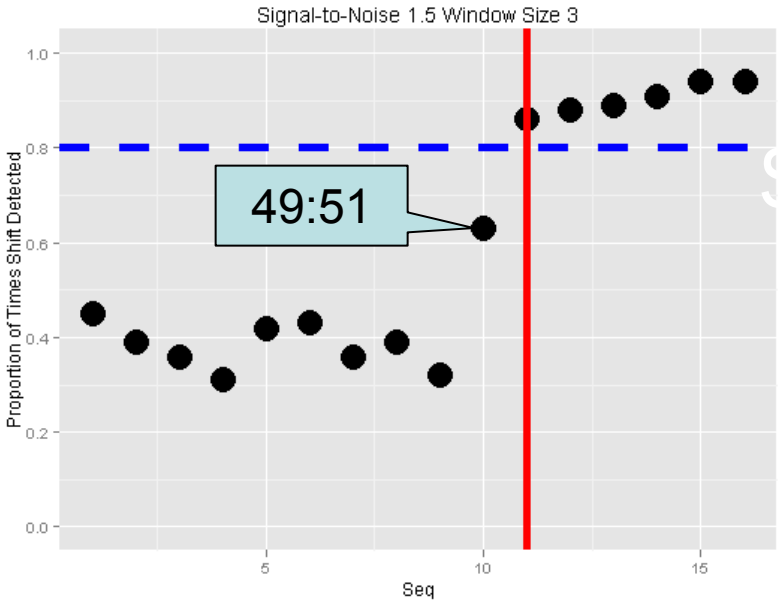




S/N=1.5, Wind=5







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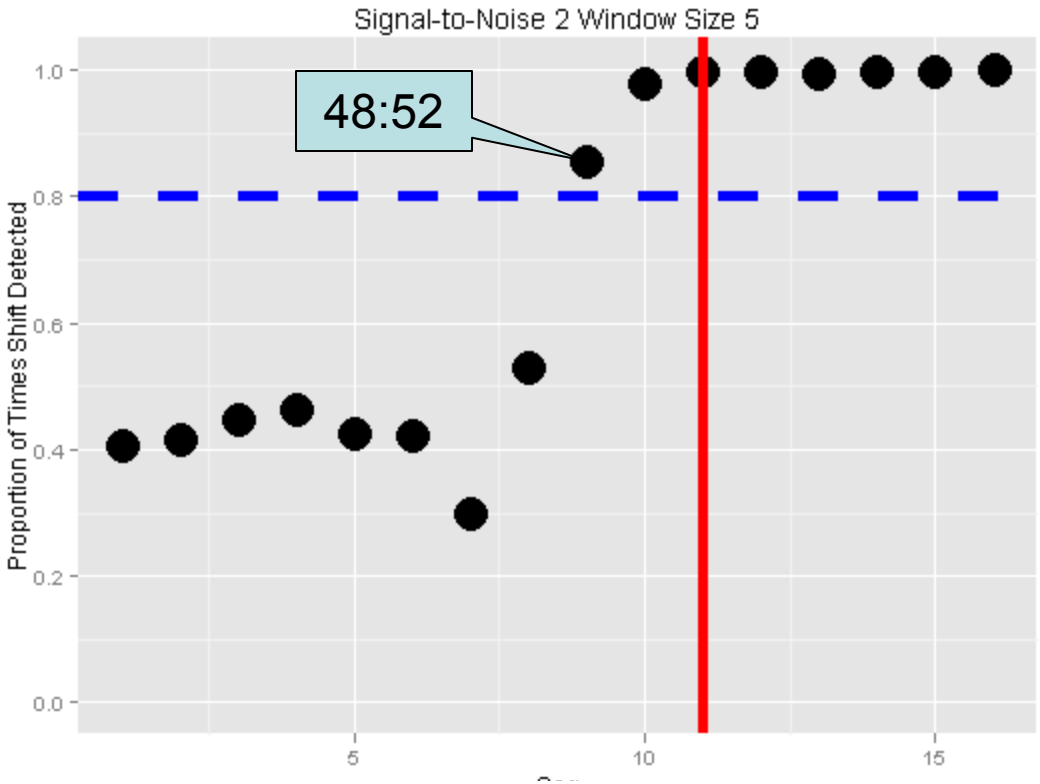
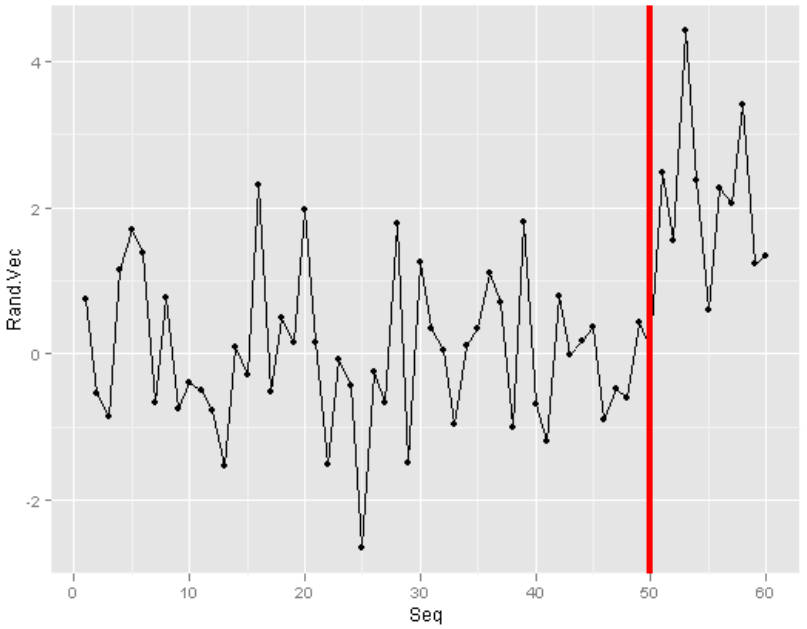
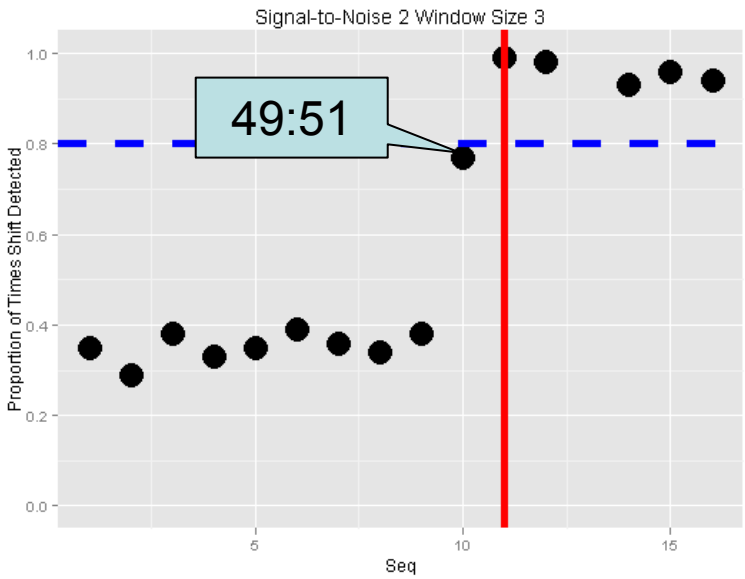
S/N=2, Wind=5

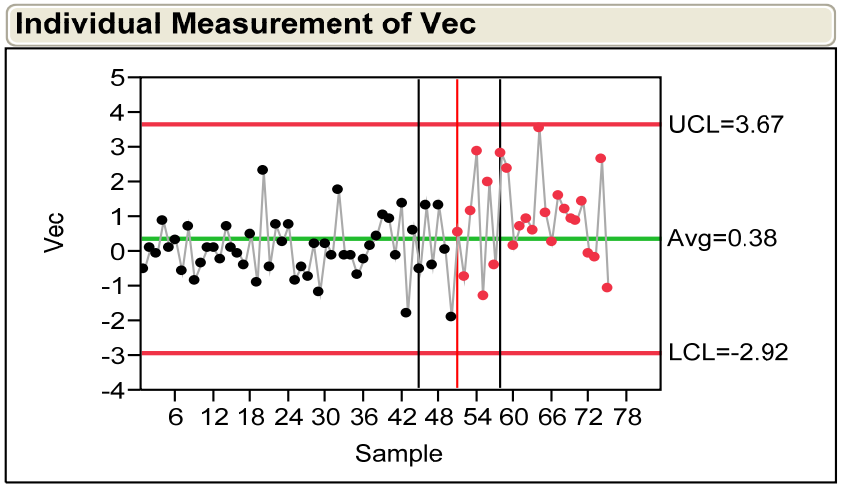


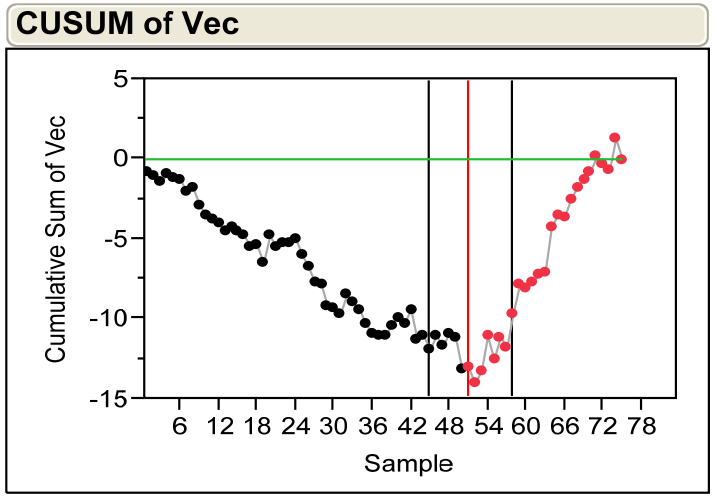




Integration of JMP® and R

Sequential Permutation Test
Simulated Process

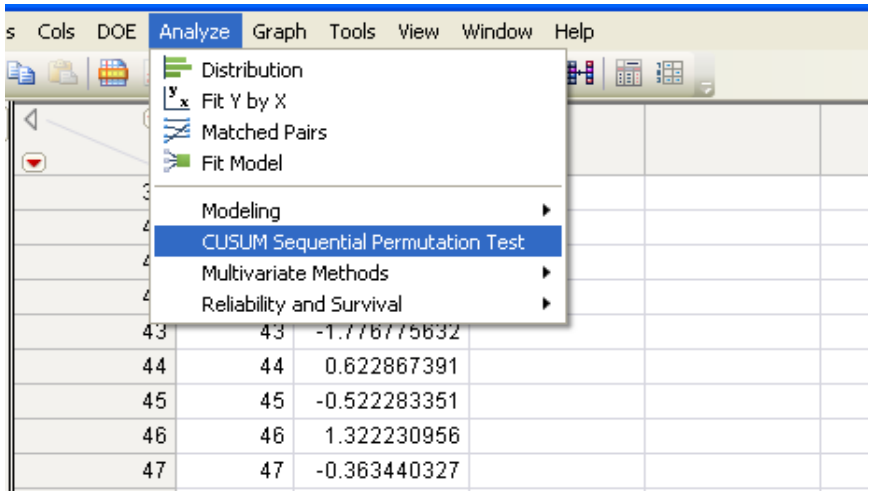


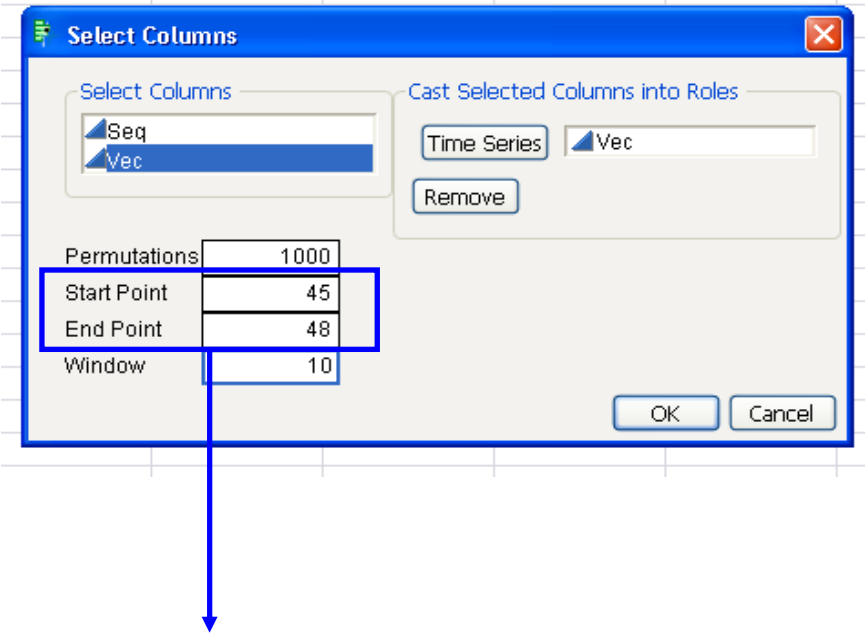


Assumption: Normal distributed data with no auto-correlation

Integration of JMP® and R

Sequential Permutation Test GUI

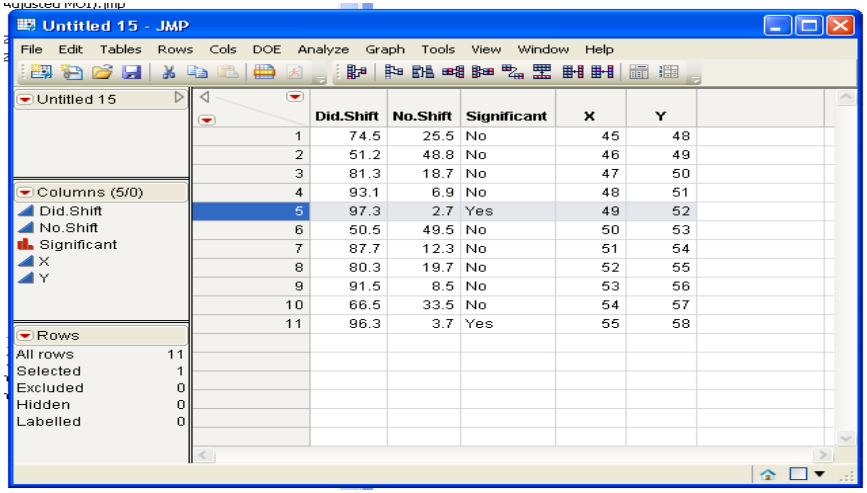




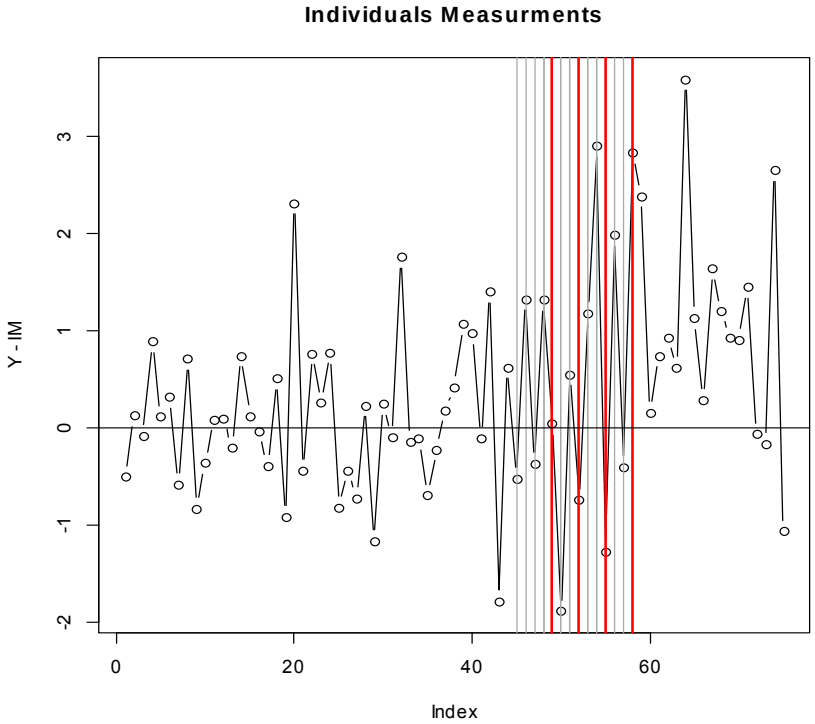
Window Size = 3

Integration of JMP® and R

Sequential Permutation Test Output
& Results w/ Actual Output



	Did.Shift	No.Shift	Significant	X	Y
1	74.5	25.5	No	45	48
2	51.2	48.8	No	46	49
3	81.3	18.7	No	47	50
4	93.1	6.9	No	48	51
5	97.3	2.7	Yes	49	52
6	50.5	49.5	No	50	53
7	87.7	12.3	No	51	54
8	80.3	19.7	No	52	55
9	91.5	8.5	No	53	56
10	66.5	33.5	No	54	57
11	96.3	3.7	Yes	55	58



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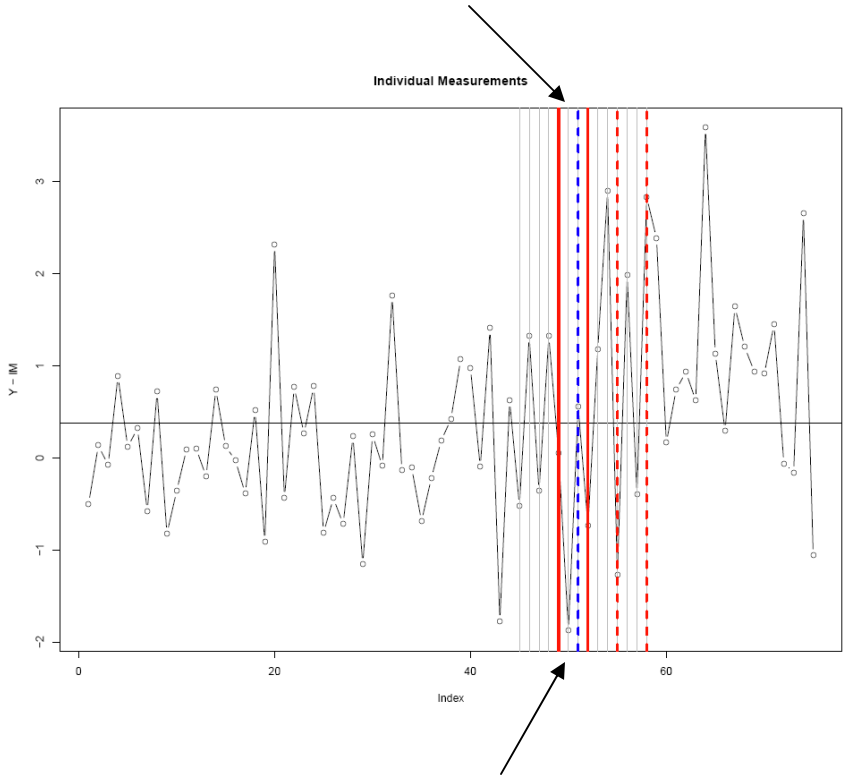
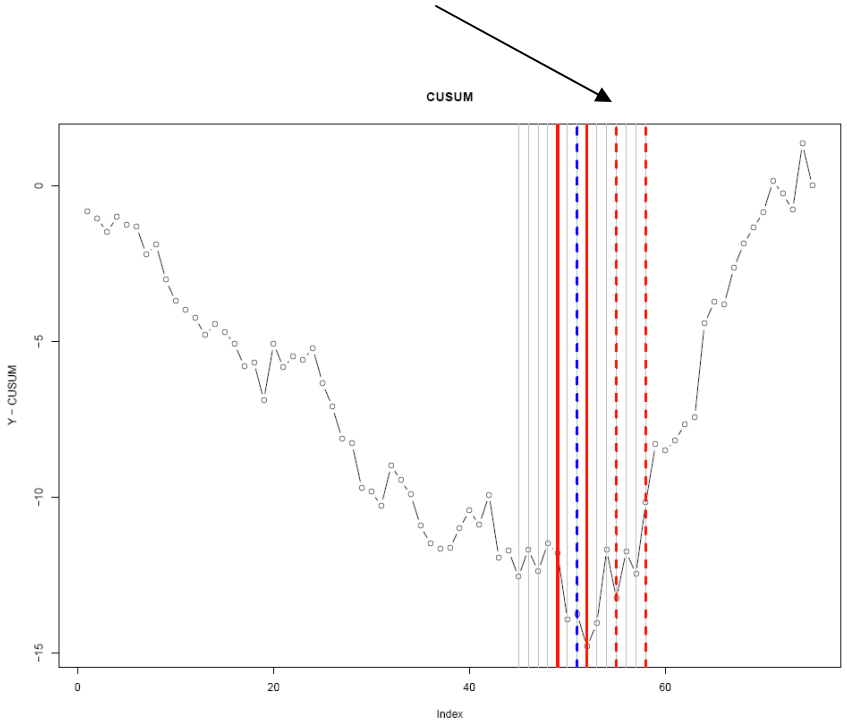
Simulated Case Study Results

Red solid lines = shift point range indentified



Red dashed lines = shift point range indentified



Blue dashed line = potential shift

Conclusions

- Change-point problem is general and can be applied in many applications such as degradation curves and linear/non-linear trends, in addition to time series models.
- Another application in manufacturing processes includes detection of the change-point for process variance.
- It is preferred to combine both analytical and visual techniques; in addition to the process expertise; to get accurate results.
- Permutation tests can provide an objective test of where a change-point may have occurred
- Future work:
  - Apply methodology to EWMA
  - Apply to linear trends
  - Impact of auto-correlated data

References

- Fahmy, H.M. and Elsayed, E.A., Drift Time Detection and Adjustment Procedures for Processes Subject to Linear Trend. *Int. J. Prod. Research*, 2006, 3257–3278.
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