

# A JMP Script that Determines a Simultaneous 95% Bound using a k-Nearest Neighbor Approach

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- Motivate the problem
- Share some possible solutions
- Share the solution we went with
- Illustrate the solution with an example
  - Show screenshots from the JMP<sup>®</sup> Add-in
- Provide highlights from the script
- Summary



### **MOTIVATING PROBLEM**

- We are designing a device
- We need to know what is the "worst case" set of 4 resistance and 4 capacitance values that we may see.
- Worst case is defined as the low resistance / high capacitance and high resistance / low capacitance combinations.
  - Low res<sub>1</sub>, low res<sub>2</sub>, low res<sub>3</sub>, low res<sub>4</sub>, hi cap<sub>1</sub>, hi cap<sub>2</sub>, hi cap<sub>3</sub>, hi cap<sub>4</sub>
  - Hi res<sub>1</sub>, hi res<sub>2</sub>, hi res<sub>3</sub>, hi res<sub>4</sub>, low cap<sub>1</sub>, low cap<sub>2</sub>, low cap<sub>3</sub>, low cap<sub>4</sub>
- Worst case may be defined as 95% confidence or 99% confidence or ...



### **ILLUSTRATION OF THE PROBLEM**

• For demonstration purposes, let's use 1 resistance and 1 capacitance value.



• We want to know 2 things

- What is the "worst" low resistance / high capacitance combination? +
- What is the "worst" high resistance / low capacitance combination?

### MOTIVATING PROBLEM - A COUPLE OF THINGS TO CONSIDER

- We know that the 8 responses / variables are not independent.
- Each of the 8 capacitance and resistance responses may or may not follow a normal or multivariate normal distribution.
- We ask these types of questions frequently
  - We need a robust solution that is easy to use
- We tend to have relatively large data sets (at least 400 points).
- A practical solution works for us (with some statistics behind it)
  - We do not have a definition of "worst-case" (e.g., 99%)
  - Better to be a little conservative

# MOTIVATING PROBLEM – A (MADE-UP) DIFFERENT INDUSTRY EXAMPLE

- We make adjustable desks for the classroom.
- We want our desks to work for 99% of the population
- Two major dimensions go into building a desk
  - A person's height
  - A person's width
    - If we use JMP<sup>®</sup> BigClass.JMP data set (and use weight for width), we could determine height and weight bounds that capture 99% of the population.



### **CURRENT APPROACH – WHICH WE BELIEVE CAN BE IMPROVED**

1-Alpha

0.950

- We independently find the 95% (or 99% or ...) prediction bounds for resistance and capacitance.
  - Two separate, independent bounds
- The worst-case is the combination of these two numbers.

▼res 1					4	cap 1			
Predict	tion Inter	rval			⊿		tion Inter	rval	
Parameter	Future N	Lower PI	Upper Pl	1-Alpha		Parameter	Future N	Lower PI	Upper Pl
Individual	1	4.523663	4.977208	0.950		Individual	1	15.60834	16.54047



### **CONCERNS WITH CURRENT APPROACH**

- Type I error (α)
  - 95% bound for resistance 1
  - 95% bound for capacitance 1
  - 95% bound for ...
  - Overall, the confidence level is not 95%
    - Depends on the correlations of the variables
- Could make an alpha adjustment



### **CONCERNS WITH CURRENT APPROACH**

- What if we were interested in the high/high combination?
- For this example, we do not really have data near the estimated worst-case bound.
- Thus, this estimate would be considered highly conservative.
  - From a design perspective, this has a cost associated with it.



- Density Ellipses
- Found in the Fit Y by X Platform





- Density Ellipse Concerns
- What is the equation of the ellipse?
- What if you have more than 2 variables?
- Pairwise ellipses same  $\alpha$  problem
- What if interested in the high/high corner?
- Which point on the ellipse do you choose?



Bivaria	te Norma	al Ellipse	P=0.950		
Variable	Mean	Std Dev	Correlation	Signif. Prob	Number
res 1	4.750436	0.115168	-0.34843	<.0001*	370
cap 1	16.07441	0.236691			

- Principal Components
- Creates new variables (e.g., Prin 1, Prin 2)
- Such that the new variables are orthogonal
- Found in Multivariate Methods





- Principal Components Concerns
- The math is difficult when there are more than 2 variables.
- In theory, Principal Components tries to "reduce dimensionality"
  - E.g. we have 8 variables
  - But 3 main principal components



6.1398081545 • res 1 + 2.9874664861 • cap 1 + -77.1885096

### WHERE DOES THAT LEAVE US?

- Goal: Find the simultaneous worst-case bounds
  - ("high/high" or "low/high" or "low/low/low/high/high/low/high" or ...)
- We would like to use JMP<sup>®</sup> to help us solve the problem
- Needs to be able to handle 3 or more variables
- Each variable may or may not be normal
- There are expected to be some correlations between variables
- We tend to have relatively large data sets
- We may ask for a "corner" where there is no data
- An easy, practical solution may be sufficient







### **CONCEPT - K-NEAREST NEIGHBORS**

- You have a point
- Find the distance from that point to every other point
- Determine the point's k nearest neighbors (k points with shortest distances)
- Example
  - The point in red is (4.51, 15.66)
  - The point in blue is (4.67, 15.54)
  - The distance =  $\sqrt{(4.51 4.67)^2 + (15.66 15.54)^2}$



- Repeat this for every point. Then, sort by distances.
- The k=2 nearest neighbors are in pink (the 2 shortest distances)

**SCALE YOUR INNOVATION** 



## **SOLUTION: USE K-NEAREST NEIGHBORS TO FIND WORST-CASE BOUNDS**

- Idea:
  - If you have a "large" data set
  - Find the median +/- 3 std devs for each variable (could also use mean)
  - Define targeted corner(s) the low or high point for each variable
  - Find the distance from each point to the "targeted corner"
  - Sort the distances from smallest to largest.
  - For our needs\*, we take the k nearest neighbors to the "targeted corner"
    - In general, collect k neighbors that represent the desired confidence
  - Take the average of the k neighbors



- Idea:
  - Find the median +/- 3 std devs for each variable (could also use mean)
  - Define targeted corners in this example, the low/high and the high/low corner.



#### • Idea:

- Find the median +/- 3 std devs for each variable (could also use mean)
- Define targeted corners in this example, the low/high and the high/low corner.
- Next, find the k-nearest neighbors to the targeted corners.
- As an illustration, the 5 nearest neighbors are found



#### • Idea:

- Find the median +/- 3 std devs for each variable (could also use mean)
- Define targeted corners in this example, the low/high and the high/low corner.
- Next, find the k-nearest neighbors to the targeted corners.
- As an illustration, the 5 nearest neighbors are found
- Next, find the average of the k nearest neighbors



- The average of the 5 nearest neighbors appears to be a reasonable worst-case bound
  - There are data points near the worst-case bound (as designed)
  - Not too conservative
  - Didn't have to worry about the distributions or correlations.



- Let's say you were interested in the high/high and low/low worst-case bounds.
- The average of the 5 nearest neighbors appears to be a reasonable worst-case bound
  - In this example, the knearest neighbor approach is wonderful relative to our current approach (which is too conservative)



## **CHOICE OF K AND SHOULD WE AVERAGE?**

- k may be based on confidence level, sample size, and philosophy
- Example n = 1000 points. 95% confidence
  - 25<sup>th</sup> closest distance for the two corners
  - Average 23<sup>rd</sup>, 24<sup>th</sup>, 25<sup>th</sup>, 26<sup>th</sup>, 27<sup>th</sup>
- We have large sample sizes. We want to be a little bit conservative. We are not driven by 95% or 99% confidence levels.
  - \*We may average the 1<sup>st</sup> through 25<sup>th</sup> points



- Pros:
  - Do not need to know the distribution of the variables
  - Can easily handle correlated / dependent variables
  - Can easily handle multiple variables
  - There is data "close" to the solution
  - We can build a script / add-in in JMP® to perform the calculations
- Con:
  - Requires a decent sized data set (sample size depends on desired confidence level)





• User Interface

- High / Low Side
- Confidence level (which drives the k)
- Recall button for convenience
- Our Team Logo





						Num Neighbors			Neighbor			Γ
	Parameter	Count	Median	Std Dev	Side	Used	Bound	Neighbor Avg	Z-score	Neighbor Values	Neighbor Indices	
1	cap 1	370	16.0842307	0.23669112	low	5	15.3741573	15.7533605	1.398	[16.0800201163215,	[10, 80, 109, 126, 321]	
2	cap 2	370	17.0965883	0.24345246	low	5	16.3662309	16.7775085	1.311	[16.6240376055925,	[10, 80, 109, 126, 321]	
3	cap 3	370	16.6350773	0.94760356	low	5	13.7922666	15.1909094	1.524	[15.5018280414842,	[10, 80, 109, 126, 321]	
4	cap 4	370	16.2480584	1.19777908	low	5	12.6547212	14.6032812	1.373	[15.374429718801,	[10, 80, 109, 126, 321]	
5	res 1	370	4.75297882	0.11516757	high	5	5.09848153	4.82126751	0.593	[4.81324536118704,	[10, 80, 109, 126, 321]	
6	res 2	370	4.79113567	0.19302804	high	5	5.37021979	5.03521616	1.264	[5.09140581005553,	[10, 80, 109, 126, 321]	
7	res 3	370	4.65214455	0.52302605	high	5	6.2212227	5.09872891	0.854	[5.6330626662091,	[10, 80, 109, 126, 321]	
8	res 4	370	4.33644006	0.14874524	high	5	4.7826758	4.42175352	0.574	[4.4187660116568,	[10, 80, 109, 126, 321]	
	1034	510	4.55044000	0.14014524	ingn		4.1020130	4.42113332	0.514	[4.4101000110500,	[10,00,100,120,021]	

- A table is provided
- Bound = Median + or 3 \* Std Dev (targeted corner)
- Neighbor Indices -> the rows of the data that are the 5 nearest neighbors
- Neighbor Z-score -> Z-score for the Neighbor Avg
  - This helps us know if our current method is conservative or not

### **OUTPUT**

	Parameter	Count	Median	Std Day	Side	Num Neighbors	Round	Neighbor Avg	Neighbor	Neighbor Values	Neighbor Indices
	Tarameter	count	WCuldii	Stubev	Juc	USCU	Dound	Neighbor Avg	Z-SCOLE	Neighbor values	Neighbor mulces
1	cap 1	370	16.0842307	0.23669112	low	5	15.3741573	15.7533605	1.398	[16.0800201163215,	[10, 80, 109, 126, 321]
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	cap 3	370	16.6350773	0.94760356	low	5	13.7922666	15.1909094	1.524	[15.5018280414842,	[10, 80, 109, 126, 321]
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5	res 1	370	4.75297882	0.11516757	high	5	5.09848153	4.82126751	0.593	[4.81324536118704,	[10, 80, 109, 126, 321]
6	res 2	370	4.79113567	0.19302804	high	5	5.37021979	5.03521616	1.264	[5.09140581005553,	[10, 80, 109, 126, 321]
	res 3	370	4.65214455	0.52302605	high	5	6.2212227	5.09872891	0.854	[5.6330626662091,	[10, 80, 109, 126, 321]
8	res 4	370	4.33644006	0.14874524	high	5	4.7826758	4.42175352	0.574	[4.4187660116568,	[10, 80, 109, 126, 321]

• The 5 rows of data (Neighbor Indices) are selected as part of the output so it is easy to color-code the points.

- In this example, I run the add-in twice.
  - Resistance high, capacitance low
  - Resistance low, capacitance high

	res 1	res 2	res 3	res 4	cap 1	cap 2	cap 3	cap 4	
105	5.02586	4.8942381681	4.9371634472	4.3182663507	15.563	16.814560715	16.644837783	15.948	
106	4.87873	4.7509117398	5.0804679806	4.3319742289	15.803	17.122181328	15.926002033	15.825	
107	4.73546	4.373291752	4.1062805249	4.314755496	16.066	17.29324638	18.213252478	17.540	
108	4.84391	4.8247083099	4.2350969454	4.3366141629	15.972	16.988901546	18.612577591	18.651	
109	4.89900	5.0290854243	4.8457331288	4.4651151647	15.663	17.004434815	14.713915973	14.879	
110	4.98062	4.8571890774	5.5407293108	4.1381344855	15.667	17.13371853	15.261269226	13.739	
111	4.96309	4.6216338752	4.3969595969	4.4226422587	15.660	16.947239266	17.827760995	16.387	
112	4.79169	4.5151569215	5.1117486139	3.9544984814	15.735	17.607730324	16.345028756	15.956	
113	4.89484	4.95417428	5.4210958221	4.3991590768	16.086	16.934412206	17.36369678	17.381	
114	4.84213	5.2145550202	4.9588187686	4.4358926514	16.005	17.15718536	16.45133484	15.922	
115	4.83525	4.8086797868	4.4816242063	4.4010138318	15.873	17.125432668	18.071551954	17.588	
116	4.64358	4.5486822131	4.2392928911	4.143455398	16.366	17.407907924	16.879875634	14.633	
117	4.87650	4.8017485597	4.4675954313	4.2731483606	15.949	17.284079121	16.234871017	16.598	
118	4.98453	4.8912727238	5.0077706239	4.4358310453	15.765	16.99886692	17.53109205	17.622	
119	4.93301	4.4943340083	4.1406931412	4.4831475847	16.051	17.157176872	16.229581198	15.885	
120	4.73818	4.5697461016	4.556376243	4.3326254382	16.732	16.722257828	15.456124521	15.892	
121	4.92099	4.6274017071	5.2724783825	4.1826395523	15.982	17.24636493	16.032728747	15.484	
122	4.84337	4.7939199528	5.1602299294	4.2018567551	15.887	17.065464163	16.425543734	16.282	
123	4.73079	4.5618454587	4.6505673275	4.1817985561	15.963	17.574534251	18.144761694	17.252	
124	4.80956	4.4759052198	5.0016508039	4.1033954486	16.027	17.394319011	17.692529323	17.829	
125	4.84085	4.586352997	4.672548348	4.32059825	16.287	17.845443825	16.750775936	15.439	
126	4.85045	5.0192383418	5.4288749704	4.2591628111	15.815	16.867689445	15.033398794	14.562	
127	4.82179	4.9035564699	4.2262461044	4.686431501	16.016	16.99144021	16.265080302	15.795	
128	4.76475	4.6299405972	3.5651862659	4.4544255212	15.987	17.389826751	15.935040461	16.090	

OUTPUT

 Green points represent low res / high cap

 Red points represent high res / low cap



# MOTIVATING PROBLEM - A (MADE-UP) DIFFERENT INDUSTRY EXAMPLE

- If we use JMP<sup>®</sup> BigClass.JMP data set (and use weight for waist size), we could determine height and weight bounds that capture 99% of the population.
- Using the add-in (nearest neighbor approach), the high/high is (67.8,141.4) and the low/low is (54.6,72.6)



					Num Neighbors					
Parameter	Count	Median	Std Dev	Side	Used	Bound	Neighbor Avg	Neighbor Z-score	Neighbor Values	Neighbor Indices
height	40	63	4.24233849	high	5	75.7270155	67.8	1.131	[66, 67, 68, 68, 70]	[4, 30, 37, 39, 40]
weight	40	105	22.201871	high	5	171.605613	141.4	1.64	[145, 128, 128, 134, 172]	[4, 30, 37, 39, 40]
								1	·	
					Num Neighbors	;				
Parameter	Count	Median	Std Dev	Side	Used	Boun	d Neighbor A	vg Neighbor Z-sco	ore Neighbor Values	Neighbor Indices
height	40	63	4.24233849	low		5 50.2729	845 5	4.6 1	.98 [55, 52, 51, 56, 59]	[3, 5, 8, 11, 15]
weight	40	105	22.201871	low		5 38.3943	869 7	2.6 1.4	59 [74, 64, 79, 67, 79]	[3, 5, 8, 11, 15]
	Parameter height weight Parameter height weight	ParameterCountheight40weight40ParameterCountheight40weight40	ParameterCountMedianheight4063weight40105ParameterCountMedianheight4063weight40105	ParameterCountMedianStd Devheight40634.24233849weight4010522.201871ParameterCountMedianStd Devheight40634.24233849weight4010522.201871	ParameterCountMedianStd DevSideheight40634.24233849highweight4010522.201871highParameterCountMedianStd DevSideheight40634.24233849lowweight4010522.201871low	ParameterCountMedianStd DevSideNum Neighbors Usedheight40634.24233849high5weight4010522.201871high5ParameterCountMedianStd DevSideNum Neighbors Usedheight40634.24233849low0weight40634.24233849low0	Parameter         Count         Median         Std Dev         Side         Num Neighbors Used         Bound           height         40         63         4.24233849         high         57.7270155           weight         40         105         22.201871         high         171.605613           Parameter         Count         Median         Std Dev         Side         Num Neighbors         Bound           height         40         105         22.201871         high         50.2729         8000           height         40         63         4.24233849         Iow         50.2729         8000           weight         40         63         22.201871         Iow         50.2729         80.943	ParameterCountMedianStd DevSideNum Neighbors UsedBoundNeighbor Avgheight40634.24233849high575.727015567.8weight4010522.201871high171.605613141.4ParameterCountMedianStd DevSideNum Neighbors UsedBoundNeighbor AvgParameterCountMedianStd DevSideSideSideBoundNeighbor Avgheight40634.24233849Iow550.272984555weight4010522.201871Iow538.39438697	ParameterCountMedianStd DevSideNum Neighbors UsedBoundNeighbor AvgNeighbor Z-scoreheight40634.24233849high575.727015567.81.131weight4010522.201871high5171.605613141.41.64ParameterCountMedianStd DevSideNum Neighbors UsedBoundNeighbor AvgNeighbor Z-scoreParameterCountMedianStd DevSideNum Neighbors UsedBoundNeighbor AvgNeighbor Z-scoreheight40634.24233849Iow550.272984554.61.4weight4010522.201871Iow538.394386972.61.4	Parameter         Count         Median         Std Dev         Side         Num Neighbors Used         Bound         Neighbor Avg         Neighbor Z-score         Neighbor Values           height         40         63         4.24233849         high         5         75.7270155         67.8         1.131         [66, 67, 68, 68, 70]           weight         40         105         22.201871         high         5         171.605613         141.4         1.64         [145, 128, 128, 134, 172]           Parameter         Count         Median         Std Dev         Side         Num Neighbors         Bound         Neighbor Avg         Neighbor Z-score         Neighbor Values           Parameter         Count         Median         Std Dev         Side         Num Neighbors         Bound         Neighbor Avg         Neighbor Z-score         Neighbor Values           height         40         63         4.24233849         Iow         5         50.2729845         S4.6         1.98         [55, 52, 51, 56, 59]           weight         40         105         22.201871         Iow         5         38.3943869         72.6         1.459         [74, 64, 79, 67, 79]

### **SCRIPT HIGHLIGHTS - INTERFACE**

- Lines 210,214: Collect inputs
  - hLst (high side) & lLst (low side)
  - Note that only numeric variables are allowed
- Also collecting "nn\_quant"
- Lines 222-223: Recall feature





### SCRIPT HIGHLIGHTS - DATA QUALITY CHECKS: VALIDATE INPUT DATA FROM USER

- Data Quality Checks
  - Lines 261-275: Nearest neighbor percent between 0 and 100 (percent)
  - Lines 277-291: Must have selected at least one column to be high/low

// start data quality checks
if(nn_quant >= 100   nn_quant <= 0,
// send message and throw
Caption(
{200, 200},
"Nearest Neighbor Percentile must be $> 0$ and $< 100$ - typical is 99.85",
Font("Arial Black"),
Font Size(16),
TextColor("red"),
BackColor("black"),
Spoken(0)
);
Wait(7);
Caption(Remove);
throw("Nearest Neighbor Percentile must be > 0 and < 100 - typical is 99.85");
- );

277	卓	<pre>if((nitems(hdatlst) == 0) &amp; (nitems(ldatlst) == 0),</pre>
278		// send message and throw
279		Caption(
280		{200, 200},
281		"You must choose at least one column on the high or low side",
282		Font("Arial Black"),
283		Font Size(16),
284		TextColor("red"),
285		BackColor("black"),
286		Spoken(0)
287		);
288		Wait(7);
289		Caption(Remove);
290		<pre>throw("You must choose at least one column on the high side");</pre>
291	F	);

### **SCRIPT HIGHLIGHTS - KEY FUNCTION**

- sumcols loops through the passed columns; it takes either 'low' or 'high' and returns a dictionary or associative array of information with the key being the column name passed and a list of information including:
  - Data type, Modeling type
  - Count (# of points), Median, StdDev
  - Bound (Med +- 3 sigma)
  - Side (low, high) which is passed
  - Data (a matrix) the column of data
  - stdize (matrix) ((data median) / StdDev +-3)^2
    - Recenters data to +- 3 sigma (high/low) from median

314 // 3. calculate the median and sigma for each variable 315 // 4. calculate either the low corner or high corner 316 // 3./4. combined into one function (sumcols) 317 loaa = sumcols(ldatlst, "low"); 318 hiaa = sumcols(hdatlst, "high");

56	//SCRIPT FUNCTIONS
57	sumcols = function({cols, side}, {default local},
58	<pre>//cols = the column list to convert into a dictionary</pre>
59	//side = the side to calculate = "low" or "high" which is median - 3 sigma or median + 3 sign
60	//return = {data type, modeling type, count, median, stddev, bound (med +- 3 stddev -depend of
61	<pre>aa = associative array();</pre>
62	for(idx=1, idx<=nitems(cols), idx++,
63	<pre>dtype = column(cols[idx]) &lt;&lt; get data type:</pre>
64	$mtype = column(cols[idx]) \iff get modeling type:$
65	data = column(cols[idx]) << get values:
66	Summarize(
67	<pre>crat = Count(Column(cols[idx])).</pre>
68	cred = Quantile(column(cols(idd)), 0.5)
60	cated = guilting(colsing(colsidel)), 0.5);
70	(std = std bev(cordinin(cors[idx]))
70	
71	11(side = 10W)
72	bound = cmed - 3 * cstd;
/3	<pre>stalze = power((((data - cmed)/cstd) + 3),2);</pre>
74	, //else high side
75	bound = cmed + 3 * cstd;
76	<pre>stdize = power((((data - cmed)/cstd) - 3),2);</pre>
77	);
78	<pre>aa[cols[idx]] = evallist({dtype, mtype, ccnt, cmed, cstd, bound, side, data, stdize});</pre>
79	- );
80	return(aa);
81	L);



### SCRIPT HIGHLIGHTS - SUM DATA AND CALCULATE DISTANCE

- Line 332-341 sum the square of the median +- 3 sigma depending on high/low
- Line 342 calculates the distance from the 3 sigma point of the data
- Line 357 returns the rank of the distance matrix and
- Line 358 finds the points (neighbors) closest to the 3 sigma points

30 31 32	<pre>// 5. for each data point find the unit distance to the combined results from #4 // just need to sum the returned standardized matrices for each variable sumdata = 0;</pre>
33	<pre>sumcounts = 0;</pre>
34 E 35 36 37	<pre>for(idx=1, idx&lt;=nitems(ldatlst), idx++, sumdata = sumdata + loaa[ldatlst[idx]][9]; sumcounts = sumcounts + loaa[ldatlst[idx]][3]; ;</pre>
38 E 39 40 41 42 43	<pre>for(idx=1, idx&lt;=nitems(hdatlst), idx++,     sumdata = sumdata + hiaa[hdatlst[idx]][9];     sumcounts = sumcounts + hiaa[hdatlst[idx]][3]; ); sumdata = sqrt(sumdata); avgcount = floor(sumcounts / (nitems(ldatlst) + nitems(hdatlst)));</pre>

351	// 6. order the rows (neighbors) from smallest to largest - don't n
352	<pre>// 7. find the number of nearest neighbors to take (minimum of 5)</pre>
353	<pre>// 8. take 5 neighbors at the minimum</pre>
354	<pre>calc_neighbor = floor((1 - (nn_quant/100)) * avgcount);</pre>
355	<pre>num_neighbor = max(5, calc_neighbor);</pre>
356	// find the ranking of the matrix to use to find the indices where
357	<pre>mtx_rnk = ranking(sumdata);</pre>
358	<pre>min_row_vector = loc(mtx_rnk &lt;= num_neighbor);</pre>

### SCRIPT HIGHLIGHTS - CALCULATE NEIGHBORS, Z-SCORE AND RETURN

- Finally, lines 346-380 loop through the low/high side and return in a dictionary:
  - Point (values) which are closest to 3 sigma points
  - Average of the minimum neighbors
  - Z-score of the median from the avg. neighbors

363	<pre>lresultaa = associative array();</pre>
364	<pre>for(idx=1, idx&lt;=nitems(ldat1st), idx++,</pre>
365	<pre>median = loaa[ldatlst[idx]][4];</pre>
366	<pre>sigma = loaa[ldatlst[idx]][5];</pre>
367	<pre>min_neighbors = loaa[ldatlst[idx]][8][min_row_vector];</pre>
368	<pre>avg_neighbors = mean(min_neighbors);</pre>
369	<pre>zscore = abs(round((median - avg_neighbors) / sigma, 3));</pre>
370	<pre>lresultaa[ldatlst[idx]] = evallist({min_neighbors, avg_neighbors, zscore, min_row_vector});</pre>
371	- );
372	hresultaa = associative array();
373	<pre>for(idx=1, idx&lt;=nitems(hdatlst), idx++,</pre>
374	<pre>hmedian = hiaa[hdatlst[idx]][4];</pre>
375	hsigma = hiaa[hdatlst[idx]][5];
376	<pre>hmin_neighbors = hiaa[hdatlst[idx]][8][min_row_vector];</pre>
377	<pre>havg_neighbors = mean(hmin_neighbors);</pre>
378	<pre>hzscore = abs(round((hmedian - havg_neighbors) / hsigma, 3));</pre>
379	<pre>hresultaa[hdatlst[idx]] = evallist({hmin_neighbors, havg_neighbors, hzscore, min_row_vector});</pre>
380	- );

### **SUMMARY**

- Motivating problem finding the simultaneous "worst-case" bounds
  - Current solution is too conservative costing us money and time
  - Data may or may not follow the multivariate normal distribution
  - Data is not independent
  - Frequency of use requires "simple" solution preferably with JMP<sup>®</sup>
- Solution build an JMP<sup>®</sup> Add-in that is easy to use
  - Uses k nearest neighbors concepts
  - Output is easy to understand.



