Modeling Trajectories with Structural Equation Models

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JMP STATISTICAL DISCOVERY FROM SAS

Outline

- Why SEM for longitudinal analysis?
- SEM Intro (*elevator version*)
- Modeling means in SEM
  - Path Diagrams ⟷ Model Equations
- Latent Growth Curve Models
- Real Data Example
  - Anxiety and health complaints during pandemic
- Summary and references

Why SEM for Longitudinal Analysis?

- Lots of flexibility

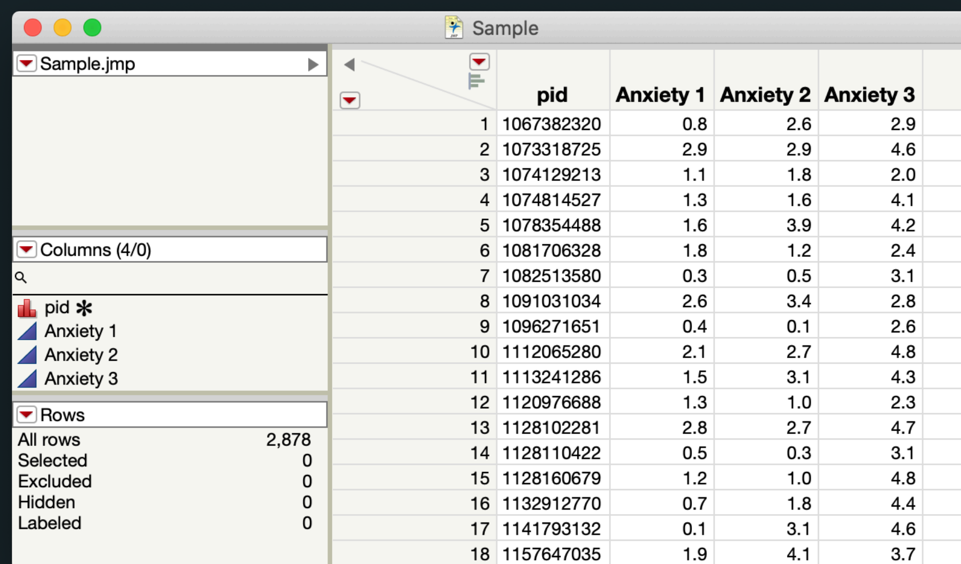
“…its [SEM’s] flexibility can dramatically extend your analytic reach.”
Singer & Willet (2003)

Why SEM for Longitudinal Analysis?

- Lots of flexibility
  - Numerous longitudinal models can be fit and compared
    - RM ANOVA, linear and nonlinear growth curve models, time series models, survival analysis*, growth mixture models*, etc.
  - Study multivariate systems
- All models profit from SEM capabilities
    - Account for measurement error explicitly (time-specific, if desired)
    - Incorporate latent variables
    - Cutting-edge algorithm for missing data
- Incorporate one's knowledge of the process under consideration

Why *not* SEM for Longitudinal Analysis?

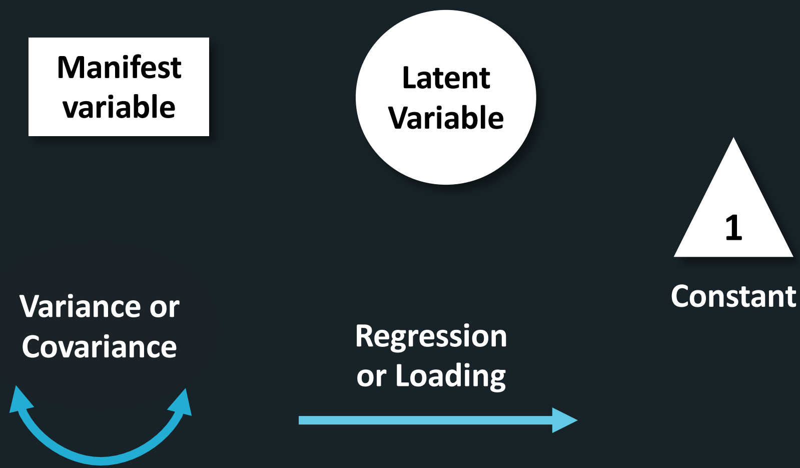
- Requires measurements at the same time-points across the sample
  - Mixed effects models more appropriate for variably spaced measurements
- Multivariate normality assumption
- Large sample technique



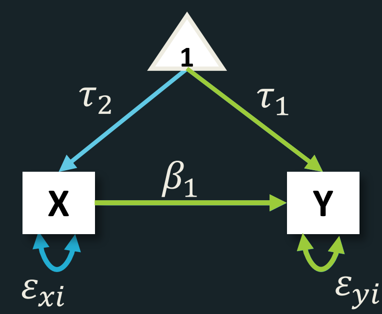
	pid	Anxiety 1	Anxiety 2	Anxiety 3
1	1067382320	0.8	2.6	2.9
2	1073318725	2.9	2.9	4.6
3	1074129213	1.1	1.8	2.0
4	1074814527	1.3	1.6	4.1
5	1078354488	1.6	3.9	4.2
6	1081706328	1.8	1.2	2.4
7	1082513580	0.3	0.5	3.1
8	1091031034	2.6	3.4	2.8
9	1096271651	0.4	0.1	2.6
10	1112065280	2.1	2.7	4.8
11	1113241286	1.5	3.1	4.3
12	1120976688	1.3	1.0	2.3
13	1128102281	2.8	2.7	4.7
14	1128110422	0.5	0.3	3.1
15	1128160679	1.2	1.0	4.8
16	1132912770	0.7	1.8	4.4
17	1141793132	0.1	3.1	4.6
18	1157647035	1.9	4.1	3.7

SEM Intro *(elevator version)*

SEM Path Diagram Elements

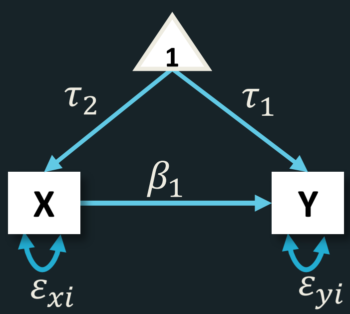


Simple Regression Example

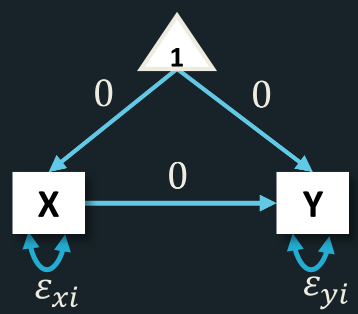


$$Y_i = \tau_1 + \beta_1 X_i + \varepsilon_{yi}$$
$$X_i = \tau_2 + \varepsilon_{xi}$$

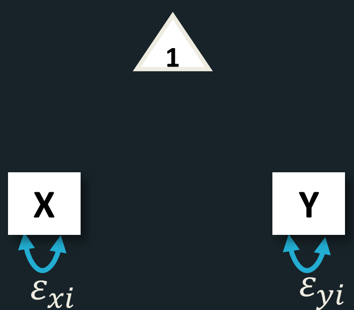
SEM Inner Workings



SEM Inner Workings



SEM Inner Workings



$$Y_i = \varepsilon_{yi}$$
$$X_i = \varepsilon_{xi}$$

implies Covariances, Means

$$\begin{array}{c} \\ X \\ Y \\ \\ \end{array} \begin{array}{cc} X & Y \\ \end{array}$$

	X	Y
X	ε_x	
Y	0.00	ε_y
	0.00	0.00

After Estimation

	X	Y
X	1.42	
Y	0.00	1.14
	0.00	0.00

Sample Statistics

	X	Y
X	1.42	
Y	0.76	1.14
	3.25	3.87

Sample minus Estimates = Residuals

	X	Y
X	0.00	
Y	0.76	0.00
	3.25	3.87

Modeling Trajectories with SEM

How anxious are you about the pandemic?

Sample

Sample.jmp

Columns (4/0)

- pid
- Anxiety 1
- Anxiety 2
- Anxiety 3

Rows

All rows	2,878
Selected	0
Excluded	0
Hidden	0
Labeled	0

	pid	Anxiety 1	Anxiety 2	Anxiety 3
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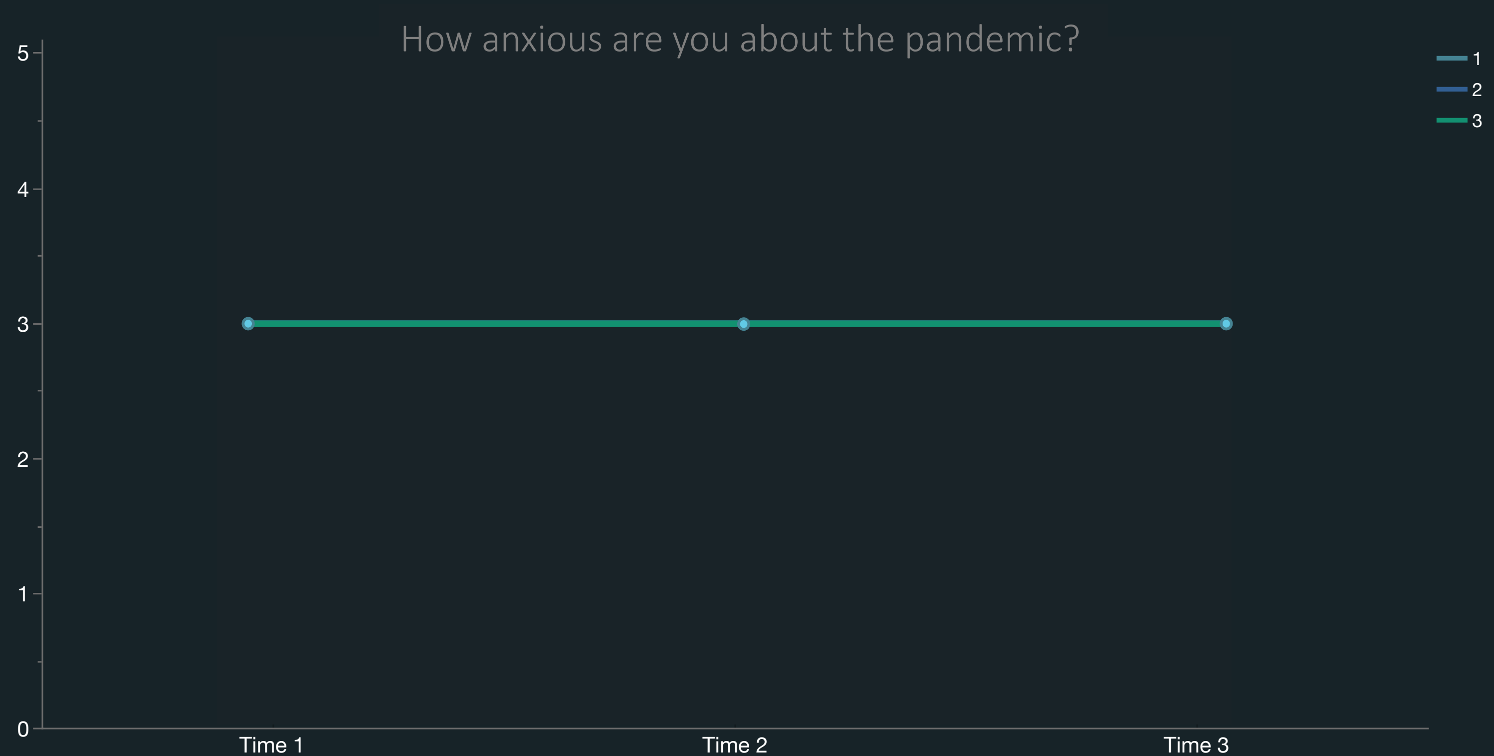
Modeling Trajectories with SEM



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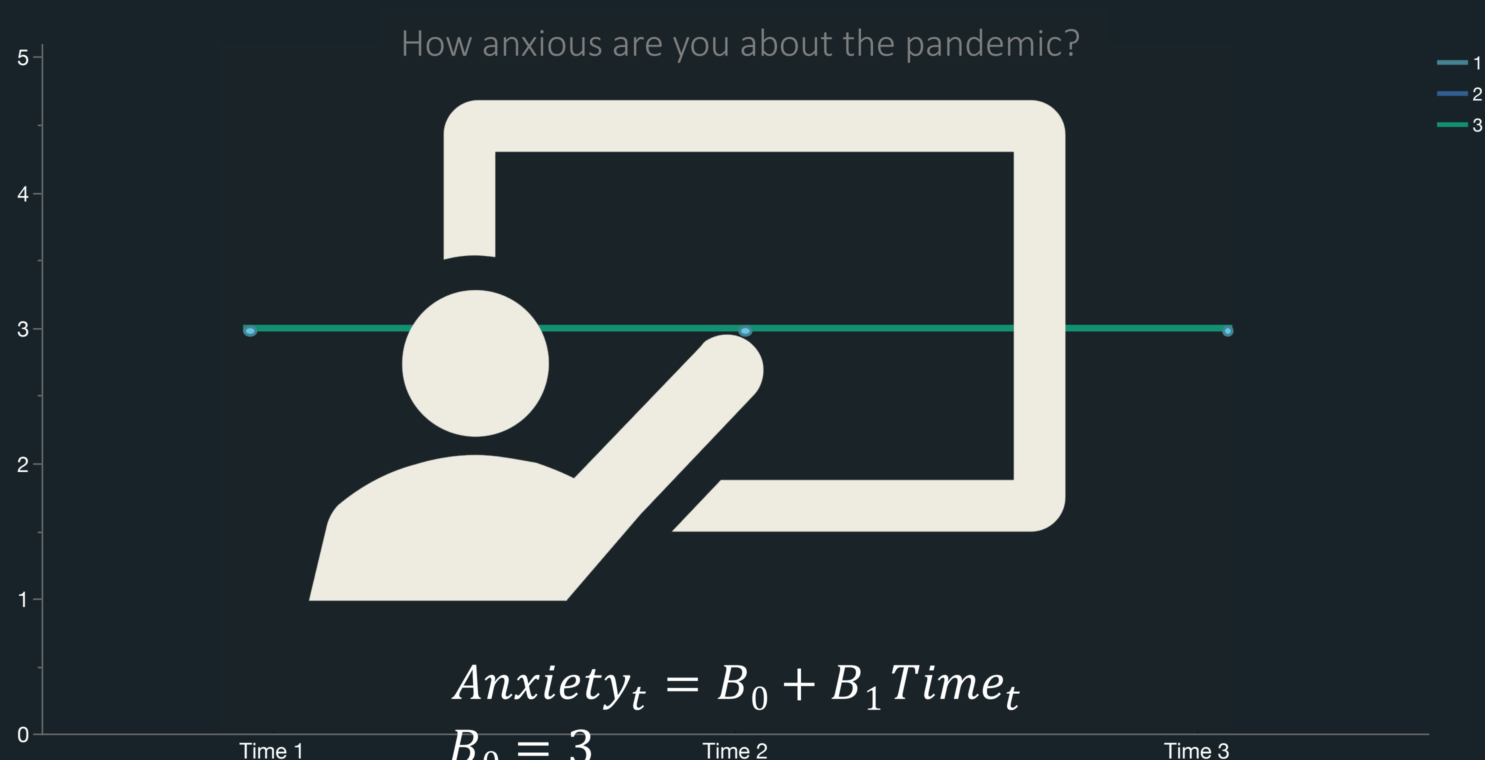
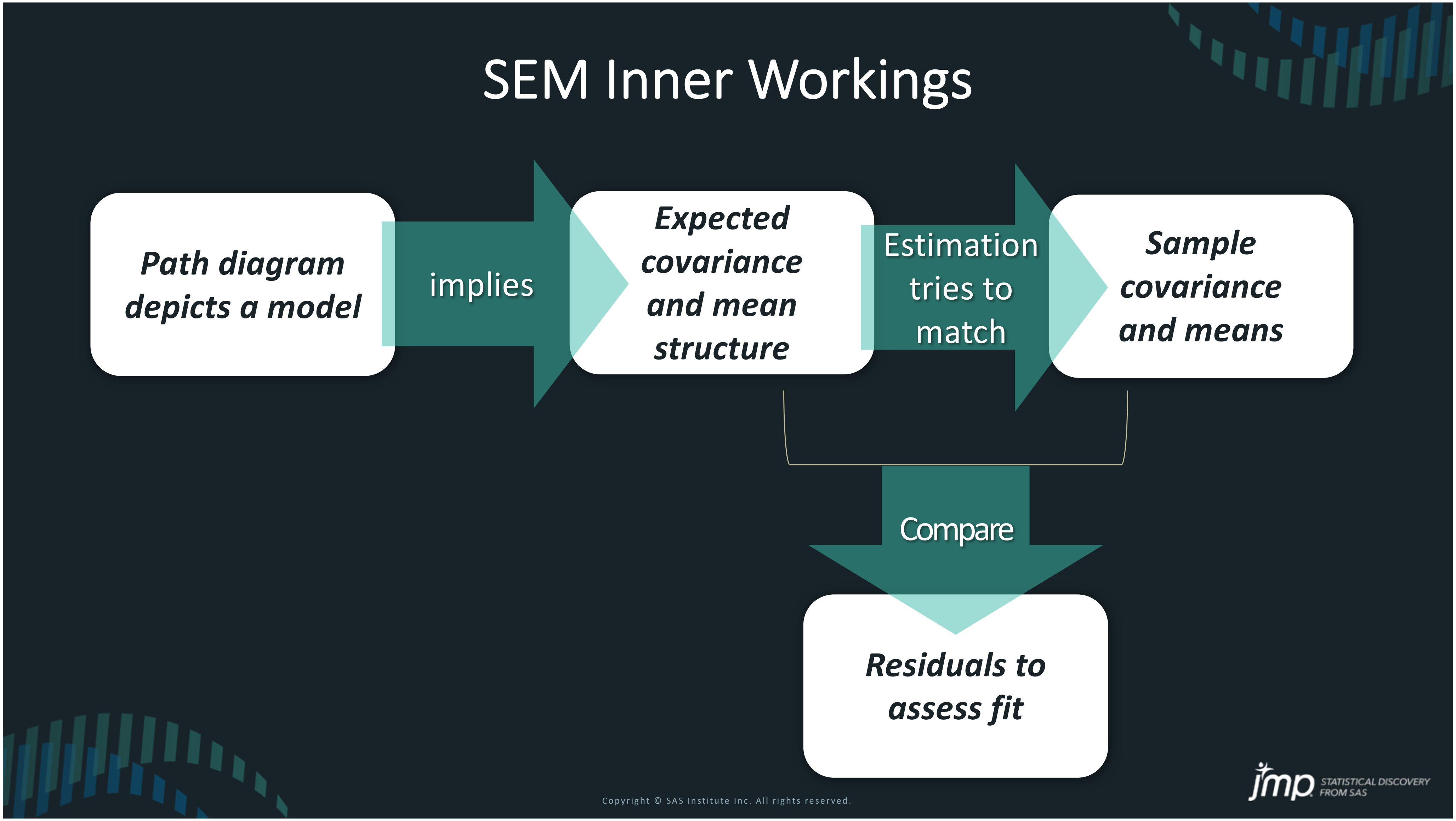
Modeling Trajectories with SEM



$$Anxiety_t = B_0 + B_1 Time_t$$

$$B_0 = 3$$

$$B_1 = 0$$


SEM Inner Workings
Path diagram depicts a model
implies
Expected covariance and mean structure
Estimation tries to match
Sample covariance and means
Compare
Residuals to assess fit
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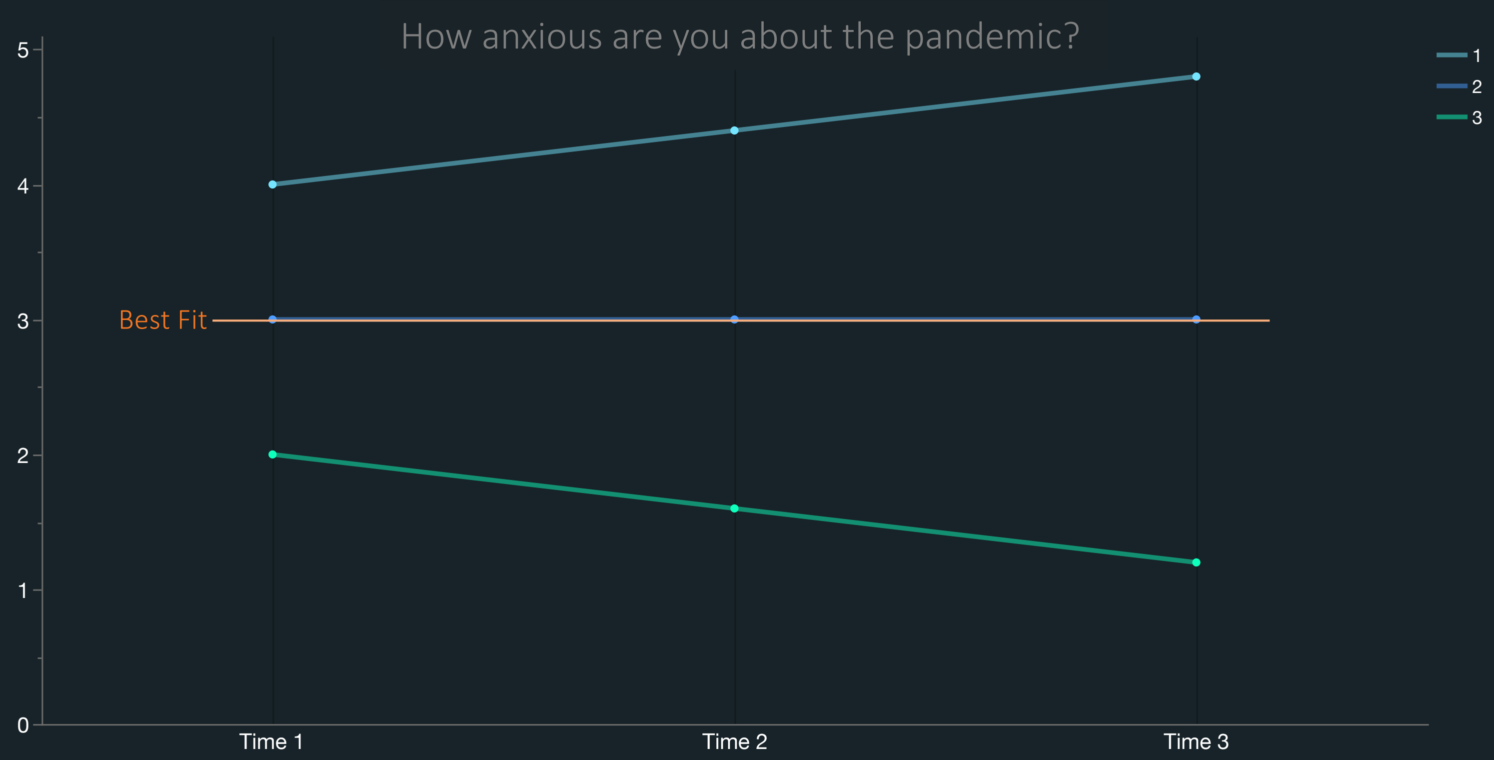
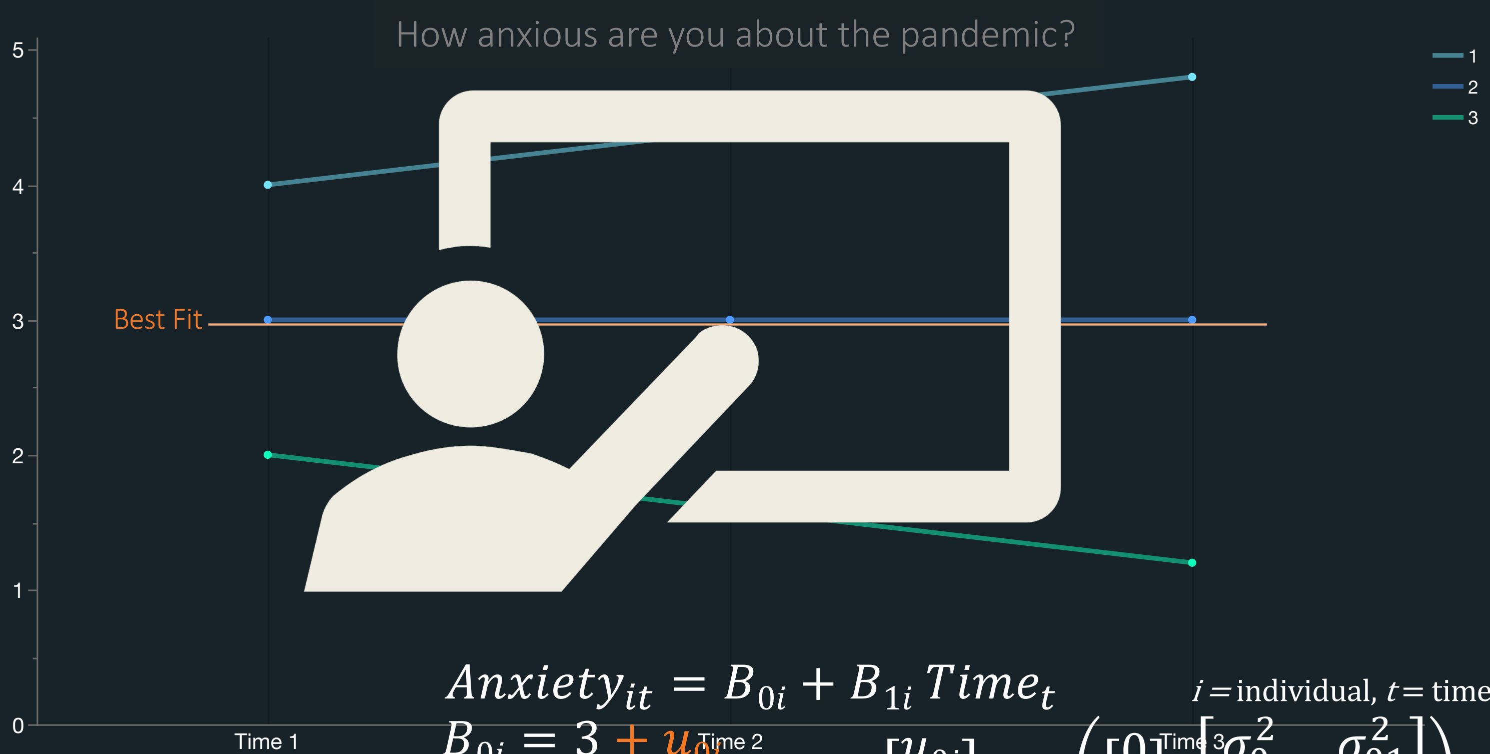
Modeling Trajectories with SEM



Modeling Trajectories with SEM



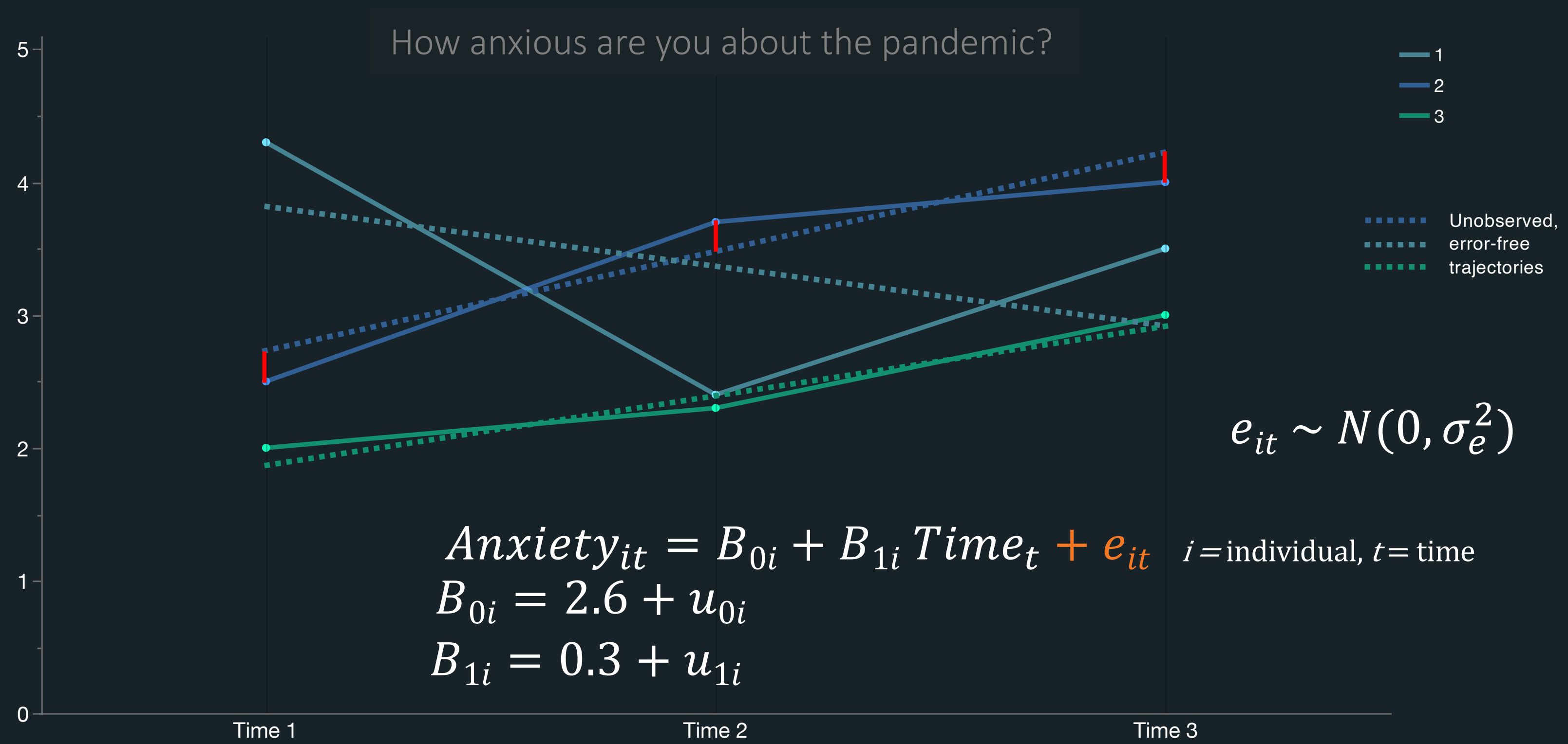
$$Anxiety_{it} = B_{0i} + B_{1i}\,Time_t \qquad i = \text{individual},\ t = \text{time}$$

$$B_{0i} = 3 + u_{0i}$$

$$B_{1i} = 0 + u_{1i}$$

$$\begin{bmatrix} u_{0i} \\ u_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01}^2 \\ \sigma_{10}^2 & \sigma_1^2 \end{bmatrix}\right)$$

Modeling Trajectories with SEM



$$e_{it} \sim N(0, \sigma_e^2)$$

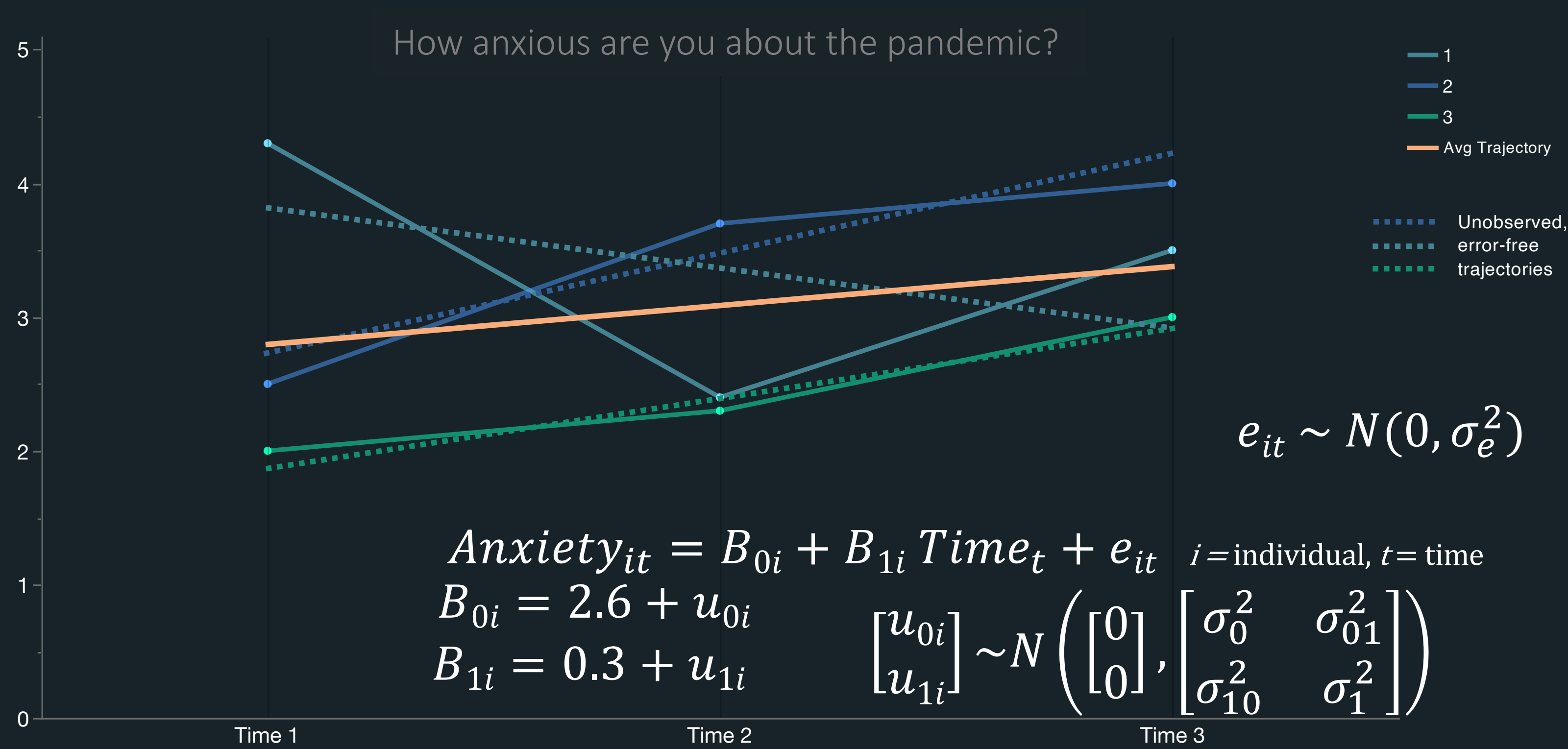
$$Anxiety_{it} = B_{0i} + B_{1i}\,Time_t + e_{it}$$ i = individual, t = time

$$B_{0i} = 2.6 + u_{0i}$$

$$B_{1i} = 0.3 + u_{1i}$$

Modeling Trajectories with SEM

Linear Latent Growth Curve Model



$$e_{it} \sim N(0, \sigma_e^2)$$

$$Anxiety_{it} = B_{0i} + B_{1i}\,Time_t + e_{it} \quad i = \text{individual}, t = \text{time}$$

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$$B_{1i} = 0.3 + u_{1i}$$

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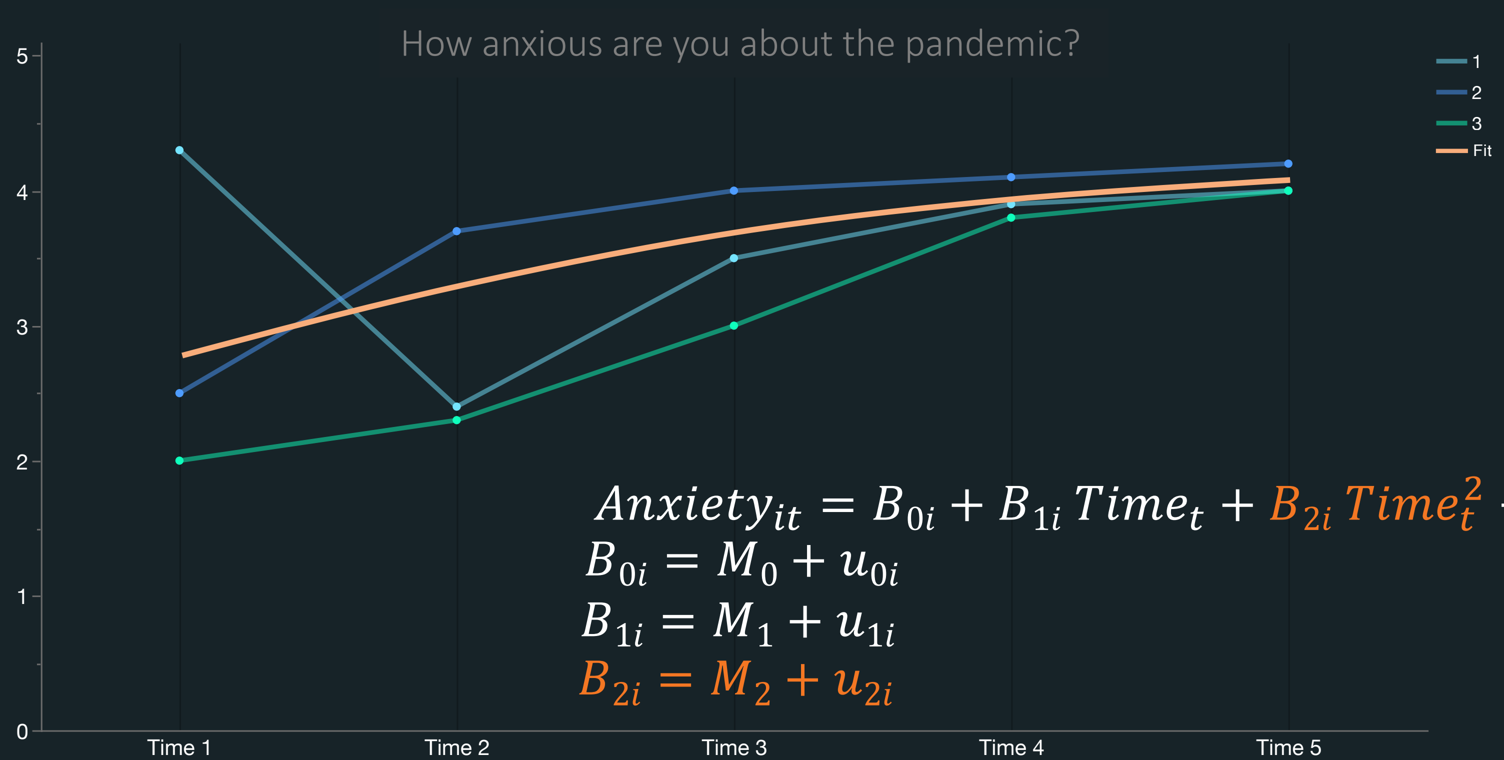
Modeling Trajectories with SEM

Quadratic Latent Growth Curve Model



$$Anxiety_{it} = B_{0i} + B_{1i}\,Time_t + B_{2i}\,Time_t^2 + e_{it}$$
$$B_{0i} = M_0 + u_{0i}$$
$$B_{1i} = M_1 + u_{1i}$$
$$B_{2i} = M_2 + u_{2i}$$

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Modeling Trajectories with SEM

Linear Latent Growth Curve Model

$$Anxiety_{it} = B_{0i} + B_{1i}\,Time_t + e_{it}$$
$$B_{0i} = M_0 + u_{0i}$$
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$$e_{it} \sim N(0, \sigma_e^2)$$

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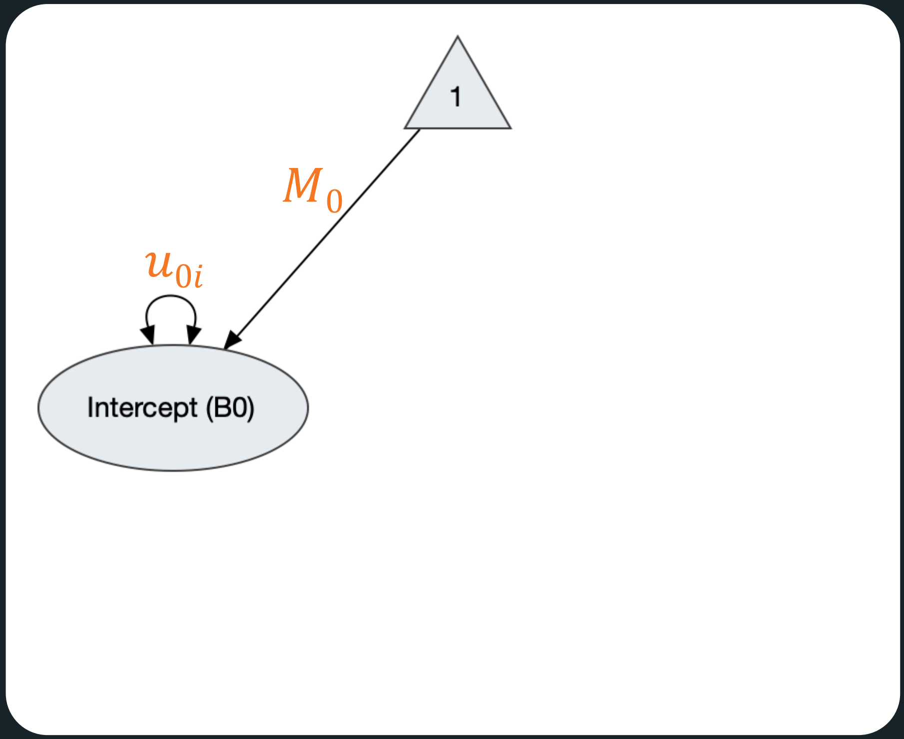

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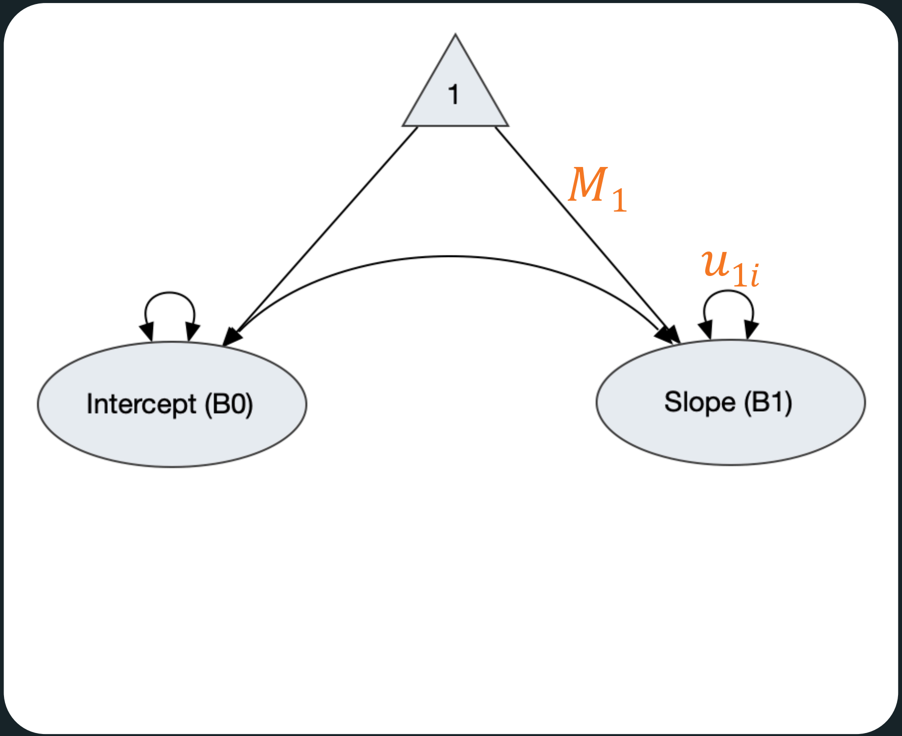

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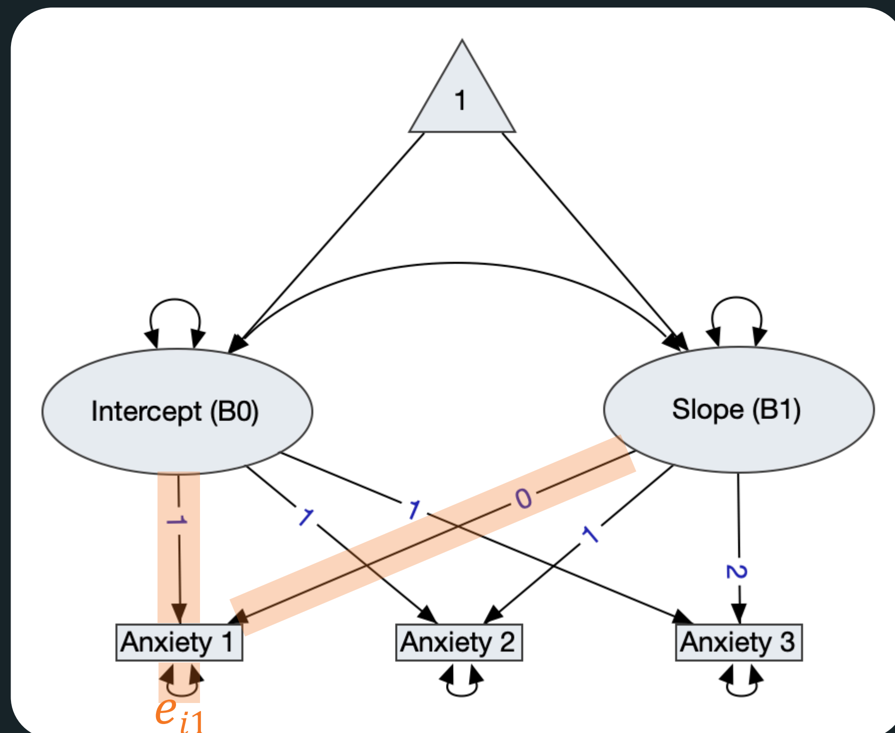
$$Anxiety\ 1_i = 1B_{0i} + 0B_{1i} + e_{i1}$$



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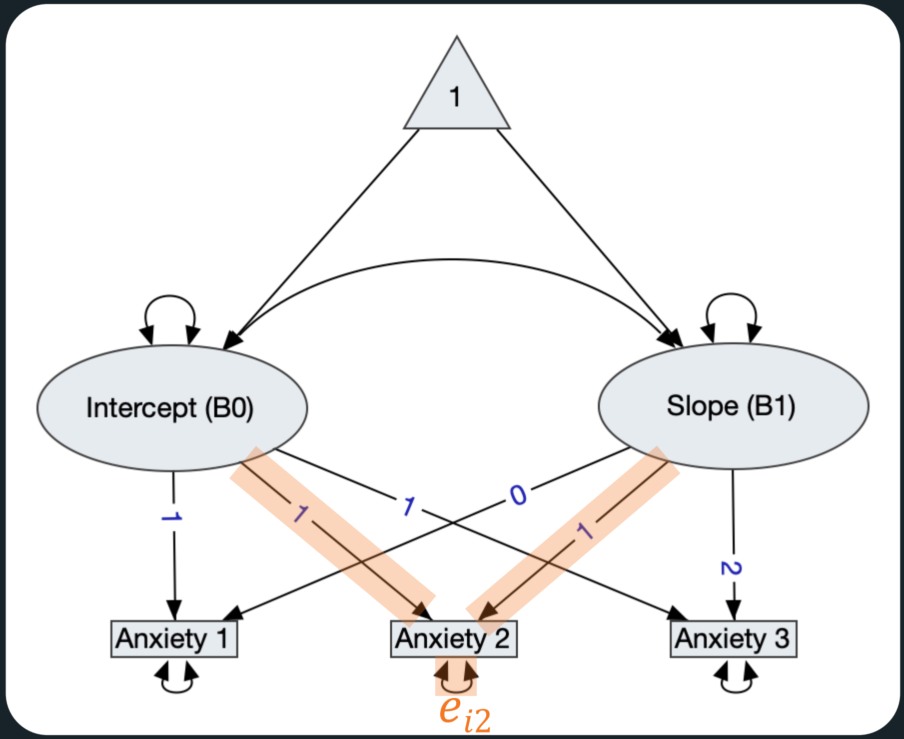
$$Anxiety\ 2_i = 1B_{0i} + 1B_{1i} + e_{i2}$$



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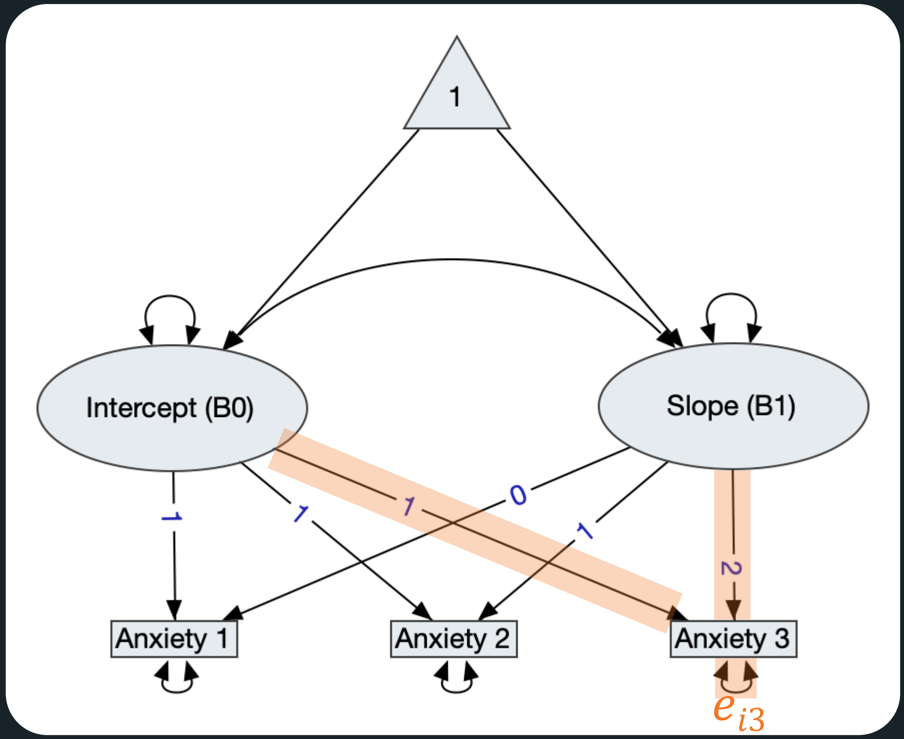

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$Time_t$

$$Anxiety\ 1_i = 1B_{0i} + 0B_{1i} + e_{i1}$$
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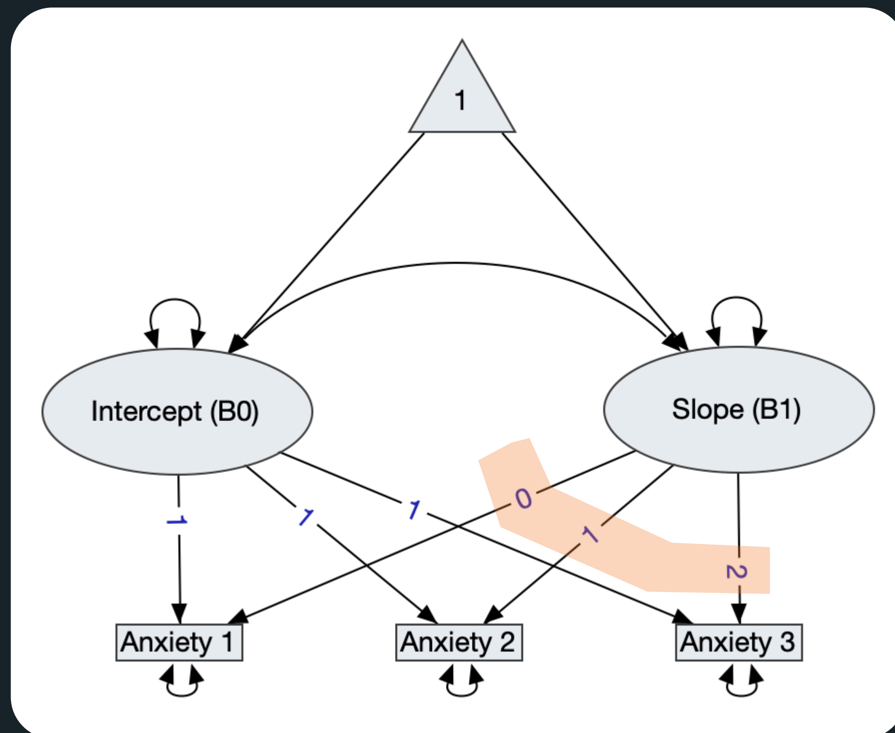

Modeling Trajectories with SEM

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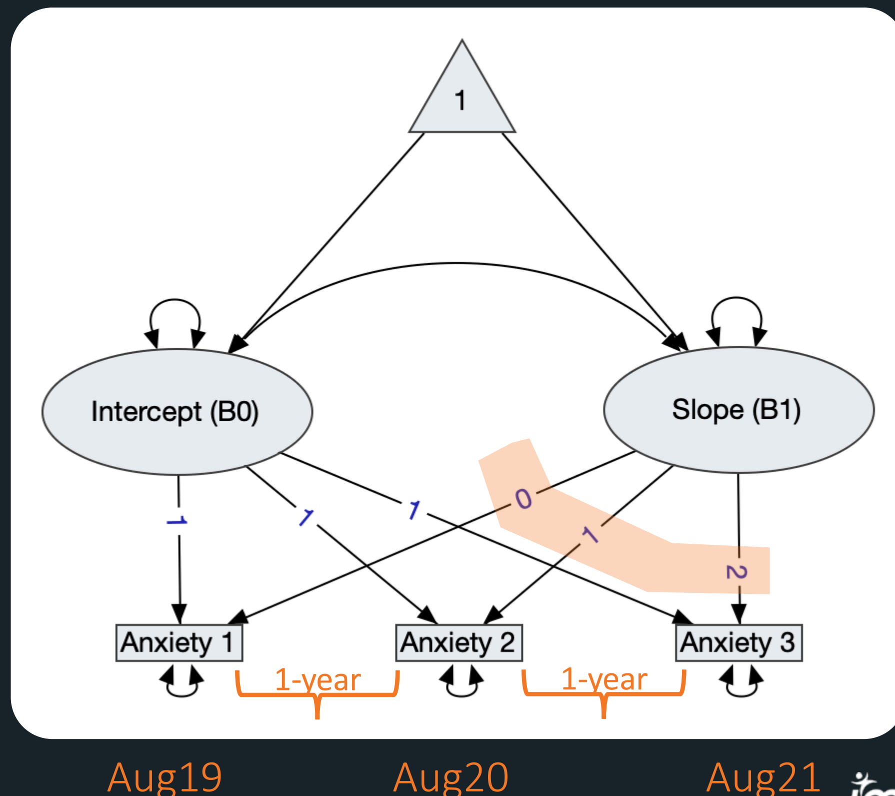

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Modeling Trajectories with SEM

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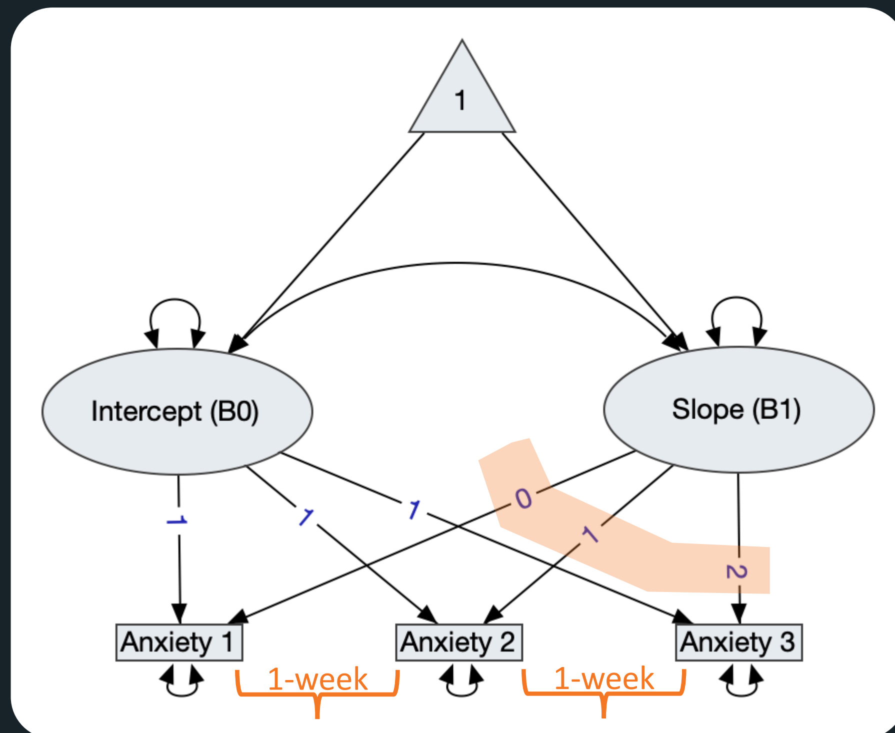

Wk1 Wk2 Wk3

Modeling Trajectories with SEM

Linear Latent Growth Curve Model

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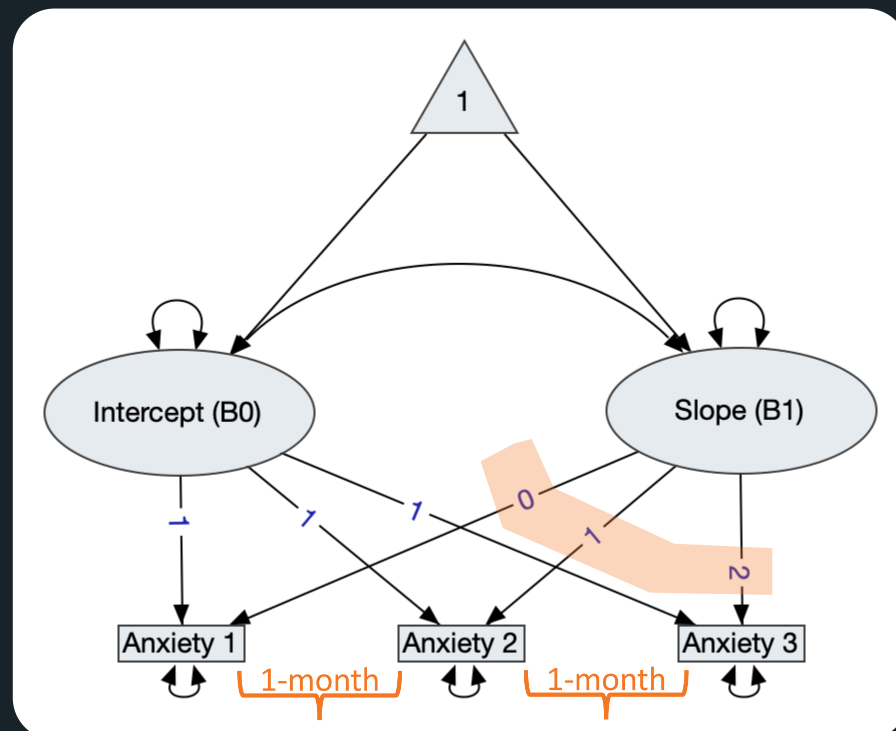

March April May

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Modeling Trajectories with SEM

Linear Latent Growth Curve Model

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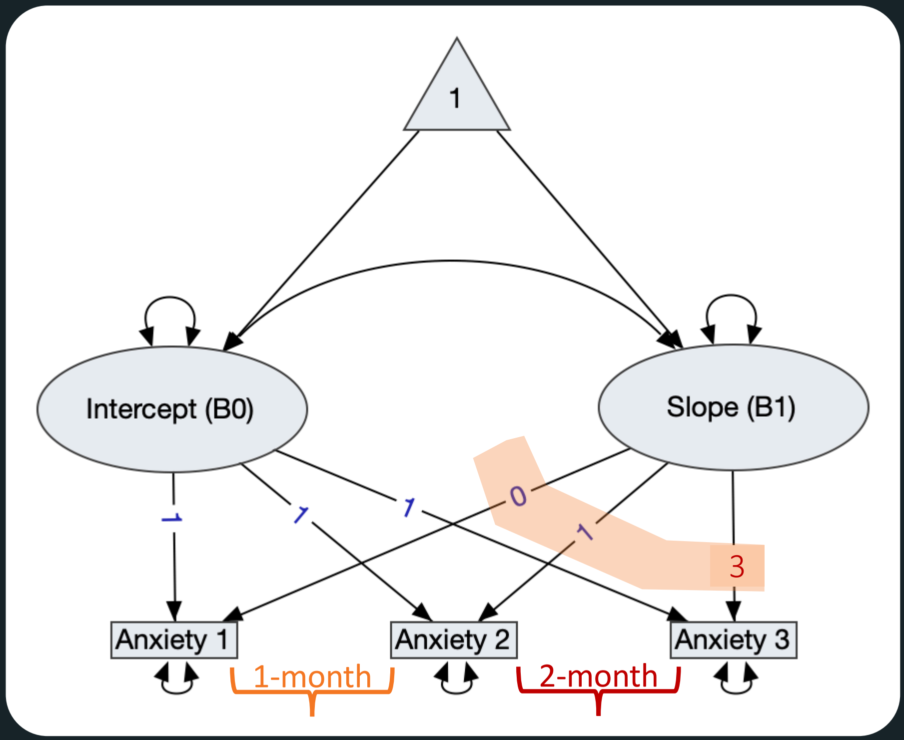
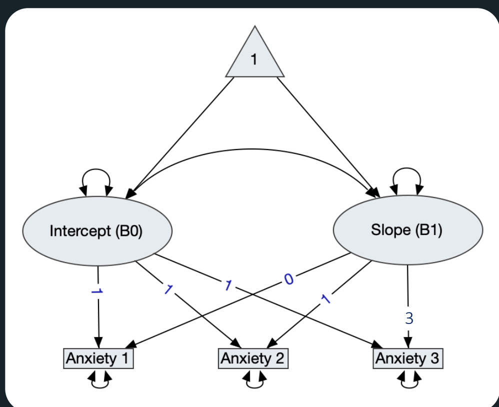
$Time_t$

$$Anxiety\ 1_i = 1B_{0i} + 0B_{1i} + e_{i1}$$
$$Anxiety\ 2_i = 1B_{0i} + 1B_{1i} + e_{i2}$$
$$Anxiety\ 3_i = 1B_{0i} + 3B_{1i} + e_{i3}$$



March April June

Implied Covariance and Mean Structures



implies Covariances

Means

$$
\begin{array}{c|ccc}
 & \textbf{Anxiety1} & \textbf{Anxiety2} & \textbf{Anxiety3} \\
\hline
\textbf{Anxiety1} & \sigma_0^2 + \sigma_e^2 & & \\
\textbf{Anxiety2} & \sigma_0^2 + \sigma_{10}^2 & \sigma_0^2 + \sigma_1^2 + 2\sigma_{10}^2 + \sigma_e^2 & \\
\textbf{Anxiety3} & \sigma_0^2 + 3\sigma_{10}^2 & \sigma_0^2 + 3\sigma_1^2 + 4\sigma_{10}^2 & \sigma_0^2 + 3\sigma_1^2 + 3\sigma_{10}^2 + \sigma_e^2
\end{array}
$$

$$
\begin{bmatrix} M_0 & M_0 + M_1 & M_0 + 3M_1 \end{bmatrix}
$$

$$Anxiety\ 1_i = 1B_{0i} + 0B_{1i} + e_{i1}$$

$$Anxiety\ 2_i = 1B_{0i} + 1B_{1i} + e_{i2}$$

$$Anxiety\ 3_i = 1B_{0i} + 3B_{1i} + e_{i3}$$

$$B_{0i} = M_0 + u_{0i}$$

$$B_{1i} = M_1 + u_{1i}$$

$$e_{it} \sim N(0, \sigma_e^2)$$

$$\begin{bmatrix} u_{0i} \\ u_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01}^2 \\ \sigma_{10}^2 & \sigma_1^2 \end{bmatrix}\right)$$

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DEMO

Repeated Measures:
Anxiety (0-100)
Health Complaints (0-28)

Time invariant:
Resilience

Three waves of data from the UK strand (*N* = 2,878)

March/2020 April/2020 June/2020

Summary

Latent Growth Curve Models

- Enable understanding of overall and individual change over time
- Identify key predictors that distinguish patterns of change
- Examine effects that growth factors have on outcomes
- Explore associations of changes across processes
- Illustration
  - Observational data ∴ No causal inferences *Experimental data can be used too!*
  - Used manifest variables for Anxiety *Latent Anxiety variables can be used too!*

RESILIENCE

Key ingredient for well-being!

References

Modeling Trajectories with SEM

JMP-Specific Video & Supplementary Material:

ABCs of Structural Equations Models
https://community.jmp.com/t5/Discovery-Summit-Americas-2020/ABCs-of-Structural-Equations-Models-2020-US-45MP-590/ta-p/281529

Journal Article:

Duncan, T. E., & Duncan, S. C. (2009). The ABC's of LGM: An Introductory Guide to Latent Variable Growth Curve Modeling. *Social and personality psychology compass*, *3*(6), 979–991. https://doi.org/10.1111/j.1751-9004.2009.00224.x

Book:

Preacher, K. J., Wichman, A. L., MacCallum, R. C., & Briggs, N. E. (2008). *Latent growth curve modeling* (No. 157). Sage.

THANK YOU

STATISTICAL DISCOVERY FROM SAS

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