Desirability Profiling and Optimization

You can define a desirability function for a single response variable or for several response variables. When you are optimizing relative to several responses, there can often be competing criteria. For example, you might want to maximize one response, minimize another, and keep a third response close to some target value.

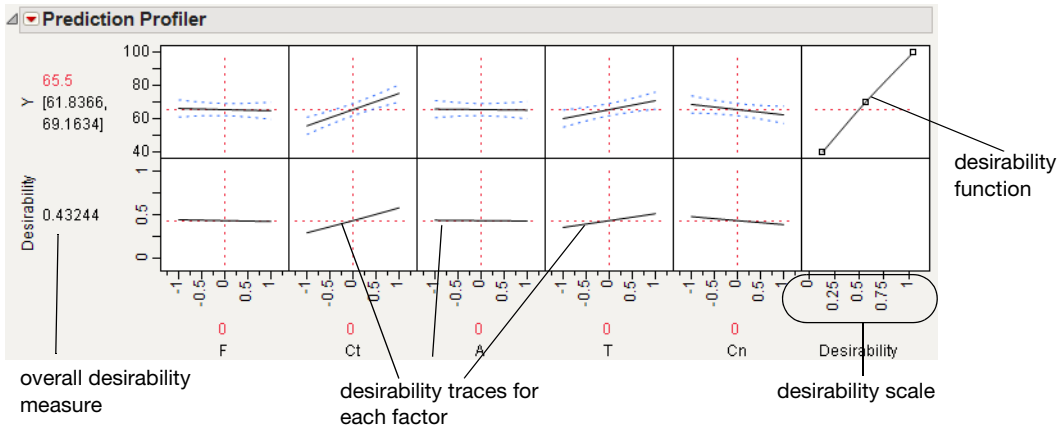
In desirability profiling, you specify a desirability function for each response. The overall desirability for all responses is defined as the geometric mean of the desirability functions for the individual responses. See Derringer and Suich (1980) for information on combining responses.

To use desirability profiling, select **Optimization and Desirability > Desirability Functions** from the Prediction Profiler red triangle menu.

Note: If the response column has a Response Limits property, desirability functions are turned on by default.

This command appends a new row to the bottom of the plot matrix, dedicated to graphing desirability. The row has a plot for each factor showing its *desirability trace*, as illustrated in Figure 3.6. It also adds a column that has an adjustable desirability function for each *Y* variable. The overall desirability measure shows on a scale of zero to one at the left of the row of desirability traces.

Figure 3.6 The Desirability Profiler



Construction of Desirability Functions

The individual desirability functions are smooth piecewise functions that are crafted to fit the control points.

- The Minimize and Maximize functions are three-part piecewise smooth functions that consist of interpolating cubics between the defining points (Low, Middle, High) and exponentials in the tails.
- The Target function is a piecewise function that is a scale multiple of a normal density on either side of the Middle value (with different curves on each side), which is also piecewise smooth and fit to the control points. Exponential functions are fit to the tails.
- The None function allows you to specify an arbitrary desirability function. In particular, you can specify desirability to be lower at the Middle value than at the Low and High values. You can also construct custom desirability functions using formulas. See "Customized Desirability Functions" on page 53.

The control points are not allowed to reach all the way to zero or one at the tail control points. This approach to constructing the desirability functions results in good behavior as the desirability values switch between the maximize, target, and minimize values.

Note: JMP does not use the Derringer and Suich (1980) functional forms. Because they are not smooth, they do not always work well with JMP's optimization algorithm.

Desirability Function for Multiple Optimization

When multiple responses are to be optimized, an overall desirability function is constructed and optimized. The overall desirability for all responses is defined as the geometric mean of the desirability functions for the individual responses.

Denote the individual desirability functions for k responses by d_1, d_2, ..., d_k. Then the overall desirability function is the geometric mean of the individual desirability functions:

$$D = d_1^{1/k} d_2^{1/k} \ldots d_k^{1/k}$$

If Importance values are defined as part of the Response Limits column property or are defined in the Response Goal window, they are integrated into the overall desirability function. The Importance values are scaled so that they sum to 1. Denote the scaled importance values by w_1, $w_{2, \ldots,} w_k$. Then the overall desirability is defined as a weighted geometric mean of the individual desirability functions:

$$D = d_1^{w_1} d_2^{w_2} \ldots d_k^{w_k}$$

Optimization Algorithm

Optimization of the overall desirability function, or of the single desirability function if there is only one response, is conducted as follows.

- For categorical factors, a coordinate exchange algorithm is used.
- For continuous factors, a gradient descent algorithm is used.
- In the presence of constraints or mixture factors, a Wolfe reduced-gradient approach is used.

How to Use the Desirability Function

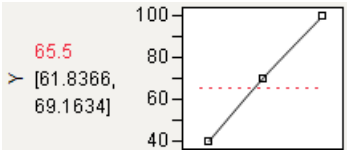
To use a variable's desirability function, drag the function handles to represent a response value.

As you drag these handles, the changing response value shows in the area labeled Desirability to the left of the plots. The dotted line is the response for the current factor settings. The overall desirability shows to the left of the row of desirability traces. Alternatively, you can select **Optimization and Desirability > Set Desirabilities** to enter specific values for the points.

Figure 3.7 shows steps to create desirability settings.

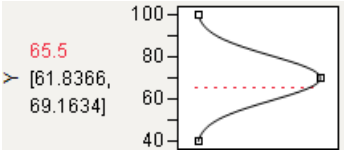
Maximize The default desirability function setting is maximize ("higher is better"). The top function handle is positioned at the maximum Y value and aligned at the high desirability, close to 1. The bottom function handle is positioned at the minimum Y value and aligned at a low desirability, close to 0.

Figure 3.7 Maximizing Desirability



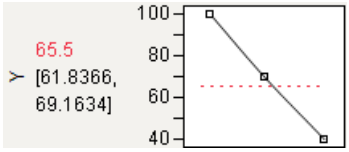
Target You can designate a target value as "best." In this example, the middle function handle is positioned at Y = 70 and aligned with the maximum desirability of 1. Y becomes less desirable as its value approaches either 40 or 100. The top and bottom function handles at Y = 40 and Y = 100 are positioned at the minimum desirability close to 0.

Figure 3.8 Defining a Target Desirability



Minimize The minimize ("smaller is better") desirability function associates high response values with low desirability and low response values with high desirability. The curve is the maximization curve flipped around a horizontal line at the center of plot.

Figure 3.9 Minimizing Desirability



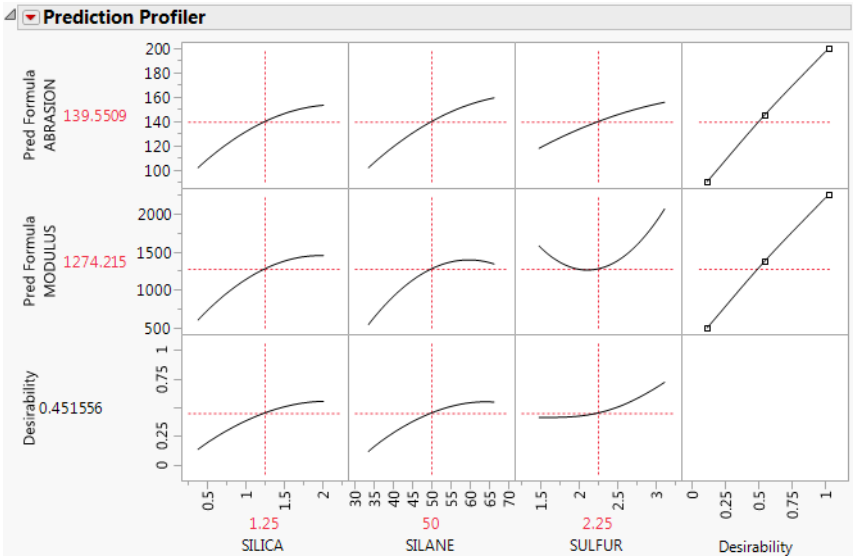
Note: Dragging the top or bottom point of a maximize or minimize desirability function across the *y*-value of the middle point results in the opposite point reflecting. A Minimize becomes a Maximize, and vice versa.

The Desirability Profile

The last row of plots shows the desirability trace for each factor. The numerical value beside the word Desirability on the vertical axis is the geometric mean of the desirability measures. This row of plots shows both the current desirability and the trace of desirabilities that result from changing one factor at a time.

For example, Figure 3.10 shows desirability functions for two responses. You want to maximize ABRASION and MODULUS. The desirability plots indicate that you could increase the desirability by increasing any of the factors.

Figure 3.10 Prediction Profile Plot with Adjusted Desirability and Factor Values



Customized Desirability Functions

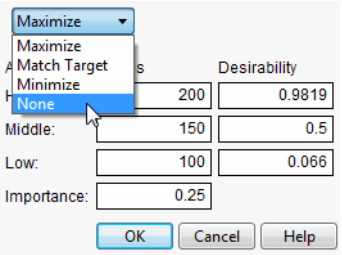
It is possible to use a customized desirability function. For example, suppose you want to maximize using the following function.

Figure 3.11 Maximizing Desirability Based on a Function

$$\frac{Pred\ Formula\ ABRASION}{96} + \frac{Pred\ Formula\ MODULUS}{700} + \frac{33}{(|Pred\ Formula\ ELONG - 450| + 1)} + \frac{2}{(|Pred\ Formula\ HARDNESS - 67| + 1)}$$

First, create a column called Custom Desirability that contains the above formula. Then, select **Graph > Profiler** to launch the platform. Select all the Pred Formula columns and the Custom Desirability column and select **Y, Prediction Formula**. Turn on the desirability functions by selecting **Optimization and Desirability > Desirability Functions** from the red triangle menu. All the desirability functions for the individual effects must be turned off. To do this, first double-click in a desirability plot window, then select **None** in the window that appears (Figure 3.12). Set the desirability for Custom Desirability to be maximized.

Figure 3.12 Selecting No Desirability Goal



At this point, selecting **Optimization and Desirability > Maximize Desirability** uses only the custom Custom Desirability function.

Figure 3.13 Maximized Custom Desirability