Shewhart's Charts and The Probability Approach

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The control chart was created by Walter Shewhart and described in his 1931 book, *Economic Control of Quality of Manufactured Product*. This technique has been proven the world over as a simple and effective means of understanding data from real-world processes. However, as one reads about "control charts" in today's technical journals, it does not seem that one is reading about the same technique that was described by Shewhart. While there is a superficial resemblance, the substance and thrust of Shewhart's charts are totally, and profoundly, different from those of the academic version of control charts. It is the purpose of this paper to examine some of these differences.

Shewhart's Control Charts

The basic concept behind Shewhart's control chart is the distinction between two categories of variation. A process will either display "controlled variation" or it will display "uncontrolled variation." If it displays controlled variation then, according to Deming, "it will not be profitable to try to determine the cause of individual variations." When a process displays controlled variation its behavior is indiscernible from what might be generated by a "random" or "chance" process (i.e. tossing coins or throwing dice). And if one is tossing, say, 100 coins, is it worth trying to find out why we get a particular number of Heads on one particular occasion? Notice the interpretation of "random" or "chance" in this concept—rather than "totally unpredictable" these terms imply only "unpredictable" in the sense of being able to predict the future *exactly*. They do imply "predictable" as regards broad characteristics of behavior—think again of tossing coins or throwing dice. When a process displays controlled variation the individual variations may be thought of as being created by a constant system of a large number of "chance causes" in which no cause produces a predominating effect.[1]

On the other hand, when a process displays uncontrolled variation, then "it will be profitable to try to determine and remove the cause of the uncontrolled variation." Here something markedly different has occurred from what would have been expected from a random or chance process, and therefore an "assignable cause" can be attributed to that occurrence. Also, seeing its effect has been large enough to notice, it is surely worth the effort to try to identify this assignable cause.

Given this distinction, the control chart is a technique for detecting which type of variation is displayed by a given process. The objective is to give the user a guide for taking appropriate action—*to look for assignable causes* when the data display uncontrolled variation, and *to avoid looking for assignable causes* when the data display controlled variation.

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[1] Shewhart, *Economic Control of Quality of Manufactured Product*, 1931, page 151.

Shewhart looked upon the control chart as the voice of the process—one can use the chart to understand how a process is behaving. Therefore, a process is said to be "in control" only when it has demonstrated, by means of a reasonable amount of data, that it is performing consistently and predictably. Moreover, from a practical perspective, such a process is, in essence, *performing as consistently as possible*.[2] Therefore, any process which is not performing "as consistently as possible" may be said to be "out of control." Thus, the control chart is a procedure for characterizing the behavior of a process.

Since the decision described above is an *inductive* inference it will always involve some degree of uncertainty. One of two mistakes can occur when decisions of this sort are made. The first mistake is to attribute an outcome to an assignable cause when it is simply the result of common causes. The second mistake is to attribute an outcome to common causes when it is, in truth, the result of an assignable cause. It is impossible to avoid both of these mistakes. So this cannot be the objective. Instead, a realistic objective is *to minimize the overall economic loss* from making these two mistakes. To this end, Shewhart created the control chart with three-sigma limits. Shewhart's use of three-sigma limits, as opposed to any other multiple of sigma, did not stem from any specific mathematical computation. Rather, Shewhart said that 3.0 "seems to be an acceptable economic value," and that the choice of 3.0 was justified by "empirical evidence that it works." This pragmatic approach is markedly different from the more strictly mathematical approach commonly seen in the journals today. In fact, in order to have a practical and sensible approach to the construction of control charts, Shewhart deliberately avoided overdoing the mathematical detail.

Notice what the control charts do—they seek to identify if the process is behaving one way or another. This, in effect, is the same as asking if the process exists as a well-defined entity, where the past can be used to predict the future, or if the process is so ill-defined and unpredictable that the past gives no clue to the future. As Shewhart said, "*We are not concerned with the functional form of the universe, but merely with the assumption that a universe exists*."

We are not trying to find some exact model to describe the process, but we are trying to determine if the process fits (at least approximately) a very broad model of "random behavior." This reversal of the roles of "process" and "model," and the broad rather than the precise nature of the "model," are not addressed in the current technical articles. Many of the papers which propose "alternatives" to control charts are focused on finding detailed models to describe the current process. While this may make sense with time series over which one has no control, it is less useful when used with processes which one can change. The model-fitting approach can provide insight into the components of, say, a time series for some economic indicator, but such a model is merely a descriptive tool. On the other hand, a production process may be changed. It may be improved. The question is how to go about the job of improving it. By comparing the process to a broad model of "random behavior" one can make the appropriate choice for process improvement. The emphasis is not upon the use of the model, but upon the characterization of the process behavior.

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[2] "This state of control appears to be, in general, a kind of limit to which we may expect to go economically in finding and removing causes of variability without changing a major portion of the manufacturing process as, for example, would be involved in the substitution of new materials or designs." Shewhart, *Economic Control of Quality of Manufactured Product*, page 25.

The Probability Approach to Control Charts

Unfortunately, some mathematical statisticians seized upon Shewhart's profound yet simple ideas and filled in what they perceived to be the mathematical gaps, and hence fell into the trap which Shewhart had been careful to avoid—the trap of reducing the usefulness of control charts.

The problem is that, in order to enable the development of a sophisticated mathematical argument, it is usually the case that assumptions need to be made which are unduly restrictive in terms of the real world. Control charting is no exception. Indeed, in this case, the assumptions required for the mathematical approach beg more than the important questions which Shewhart set out to answer. Even more unfortunately, this weakened version (though it is often perceived as stronger due to its mathematical rigor) has spread via the technical journals and is essentially the only version known to many in academic circles.

Exact mathematical methods are both easier to teach and more impressive to the unwary. But they have seriously reduced the potential of what may be accomplished through the use of control charts. And just what is the "probability approach?" The common version uses control limits which are calculated so that, supposedly, while the process is in control, there is some small chance (often 13.5 chances in 10,000) that a point will lie above the upper control limit, and the same small chance that it will lie below the lower control limit. To understand the fallacy of this approach it is instructive to consider Shewhart's argument on pages 275-277 of *Economic Control of Quality of Manufactured Product*. On these pages Shewhart seems to toy briefly with the idea of using the probability approach. Paraphrasing his argument on these pages, he points out that *if* a process were *exactly* stable, i.e. did unwaveringly fit some precise mathematical model, and *if* we knew the details of its underlying (fixed) statistical distribution, we *could* then work in terms of probability limits. However, he notes that, in practice, we never know the appropriate type of statistical distribution. While statisticians usually plump, almost as if it is a foregone conclusion, for their favorite distribution, the Gaussian or *normal* distribution, Shewhart disposes of the use of the normal distribution on page 12 of his 1939 book, *Statistical Method from the Viewpoint of Quality Control*. And in the 1931 book he notes that, even it the process were exactly stable, and if the normal distribution were appropriate (neither of which we would ever know), we would still never know the value of its mean. And even if we did, we would never know the value of its standard deviation. We could only estimate these from the data. Since the probability calculations depend upon *all* of these things, it will always be impossible in practice to compute the required probabilities.

Furthermore, in *Out of the Crisis,* Deming points out that, in practice as opposed to mathematical theory or hypothesis, *exactly stable processes never exist*. Real processes are *never* entirely free of assignable causes. This identifies a vast gulf between the common mathematical assumptions and the real world. What does this imply? Surely not that we are to spend all of our time looking for assignable causes! No, of course not. What we need is guidance as to *when assignable causes are troublesome enough to warrant attention*. Shewhart's control chart, with its three-sigma limits, provides this guidance. No calculations from the normal distribution, or any other distribution, were involved in the choice of the multiplier of 3.0. Certainly, Shewhart did then check that this multiplier turned out to be reasonable under the artificial conditions of a normal distribution— and plenty of other circumstances as well. But that is a far cry from deducing that the choice of 3.0 was *based* on the assumption of a normal distribution.

On pages 334–335 of *Out of the Crisis* Deming says:

> "The calculations that show where to place the control limits on a chart have their basis in the theory of probability. It would nevertheless be wrong to attach any particular figure to the probability that a statistical signal for detection of a special cause could be wrong, or that the chart could fail to send a signal when a special cause exists. The reason is that no process, except in artificial demonstrations by use of random numbers, is steady, unwavering. It is true that some books on the statistical control of quality and many training manuals for teaching control charts show a graph of the normal curve and proportions of area thereunder. Such tables and charts are misleading and derail effective study and use of control charts."

Thus, a major problem with the probability approach to control charts is that it is totally out of contact with reality. The assumptions used for the mathematical treatment become prohibitions which are mistakenly imposed upon practice. Restrictions such as the following are commonly encountered. "The data have to be normally distributed." "The control chart works because of the central limit theorem—therefore you have to have subgroups of at least five observations." "The chart will not work with serially-correlated (autocorrelated) data—the observations must be independent of each other before you can use a control chart." These, and others like them, are examples of the tail wagging the dog. *The assumptions selected for the convenience of the mathematician are turned into prerequisites for the use of the technique*. In the case of control charts, this reversal is both inappropriate and misleading.

In other words, the control chart is not found at the end of a mathematical syllogism. It is a procedure which is (1) consistent with theory, and (2) based upon empirical evidence that it works. This empirical justification is what takes it outside the restrictions of the Probability Approach and makes it a tool for the real world. While other tools may be compared with the control chart by means of the Probability Approach, and while other tools may even be simple enough and robust enough to work in the real world, such theoretical comparisons do not justify the use of any technique in practice.

Many statisticians consider a control chart to simply be "a sequential test of hypothesis procedure with an uncontrolled Type I error rate."[3] While the control chart may be described, on a purely mechanical level, in such terms if the necessary (but unrealistic) assumptions are made, the use of the chart is completely different. Control charts are much broader than this narrow view of their nature. They examine the data for behavior compatible with the *existence* of a universe (without necessarily implying that one can or does exist), rather than merely looking for a change in one or more parameters of a well-defined universe. No probabilities can be attached to the first decision, simply because there is no probability structure without a well-

[3] While the commonly quoted error rate of 0.27% is said to apply to each individual subgroup, the overall error rate for the whole chart is said to increase without bound as subgroup after subgroup is added. (Of course these probabilities are purely theoretical and do not actually apply in practice.)

defined universe. This is why the use of the probability approach severely limits the perceived usefulness of the control chart.

This limited view of what control charts are for is the most serious drawback of the probability approach. Those having this limited view have a narrow understanding of how to use control charts. For example, some think that control charts are only useful for process *monitoring*. The probability approach does not allow for much else because there most certainly is no distribution, and no probability, when a process displays uncontrolled variation. While a control chart may be used for process monitoring, the point is that monitoring is only a minor part of what control charts can do, rather than being *all* that they can do. The probability approach generally relegates the control chart to the role of a mere monitoring procedure after the process is presumed, by some undefined means or another, to find itself in a satisfactory state. The control chart's function is perceived to be an early-warning device of when the process moves away from this supposedly satisfactory state.

The crucial difference between Shewhart's work and the probability approach is that his work was developed in the context, and with the purpose, of process *improvement* as opposed to process *monitoring*. From his perspective, a major purpose for creating the control chart was to provide help to get the process into a "satisfactory state" which one might then be content to monitor (if not persuaded by arguments for the need of continual improvement).

This difference is far more important than some might at first appreciate. It gets right to the heart of the divide between the main approaches to the whole quality issue. On the one hand, we have approaches which regard quality merely in terms of conformance to requirements, meeting specifications, Zero Defects. On the other hand, we have Deming's demand for continual improvement—a never-ending fight to reduce variation. The probability approach can only cope with the former. Shewhart's own work was inspired by the need for the latter.

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