Analyze Fatigue Test Data using Fatigue Model Platform in 18

Peng Liu, Principal Research Statistician Developer
Rajneesh Rajneesh, Sr Analytics Software Tester

March 14th, 2024

History

engineers and scientists
collecting and analyzing
fatigue test data
since 1800s

but

existing statistical approaches are archaic
for technical, conceptual, and maybe other reasons

Modern Approach

Statistics > Methodology

[Submitted on 8 Dec 2022 (v1), last revised 15 Nov 2023 (this version, v2)]

Modern Statistical Models and Methods for Estimating Fatigue-Life and Fatigue-Strength Distributions from Experimental Data

William Q. Meeker, Luis A. Escobar, Francis G. Pascual, Yili Hong, Peng Liu, Wayne M. Falk, Balajee Ananthasayanam

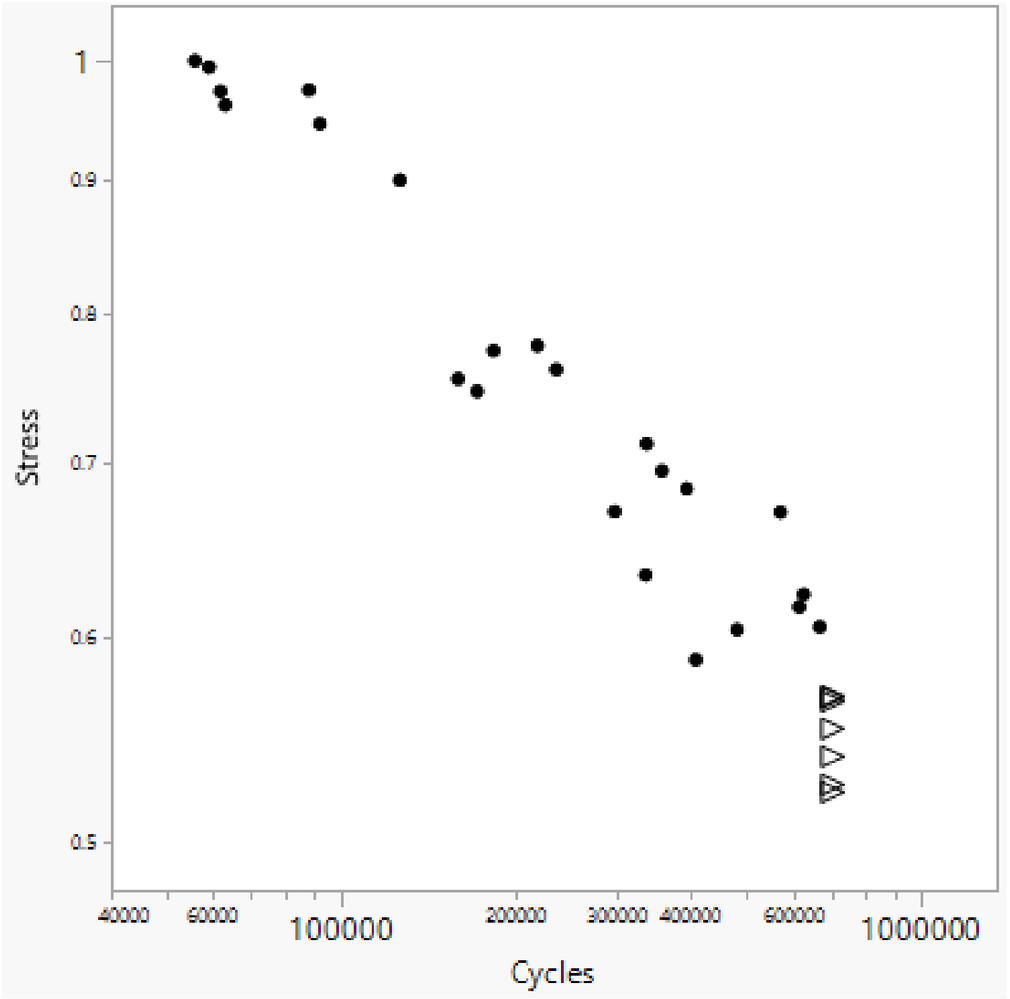
Engineers and scientists have been collecting and analyzing fatigue data since the 1800s to ensure the reliability of life-critical structures. Applications include (but are not limited to) bridges, building structures, aircraft and spacecraft components, ships, ground-based vehicles, and medical devices. Engineers need to estimate S-N relationships (Stress or Strain versus Number of cycles to failure), typically with a focus on estimating small quantiles of the fatigue-life distribution. Estimates from this kind of model are used as input to models (e.g., cumulative damage models) that predict failure-time distributions under varying stress patterns. Also, design engineers need to estimate lower-tail quantiles of the closely related fatigue-strength distribution. The history of applying incorrect statistical methods is nearly as long and such practices continue to the present. Examples include treating the applied stress (or strain) as the response and the number of cycles to failure as the explanatory variable in regression analyses (because of the need to estimate strength distributions) and ignoring or otherwise mishandling censored observations (known as runouts in the fatigue literature). The first part of the paper reviews the traditional modeling approach where a fatigue-life model is specified. We then show how this specification induces a corresponding fatigue-strength model. The second part of the paper presents a novel alternative modeling approach where a fatigue-strength model is specified and a corresponding fatigue-life model is induced. We explain and illustrate the important advantages of this new modeling approach.

Plan

- Why is the subject important?
  - Stories, Applications
- Background
  - S-N data (you should know)
  - Archaic methods (you may not realize)
- Modern approach
  - Do it right
  - Data characteristics
  - Models in Fatigue Model platform
  - List of results
- Demo

S-N Data

Stress	Cycles	Censor
0.5227282965	700000	Runout
0.5529910281	700000	Runout
0.5684164973	700000	Runout
0.6160562854	615269.06102	Failure
0.6841078509	393511.05703	Failure
0.9945327437	59144.227821	Failure
0.5878166336	407723.6345	Failure
0.7459748654	171394.39299	Failure
0.6952094588	356504.94247	Failure
1	55972.051777	Failure
0.7604927171	234686.63889	Failure
0.5261815609	700000	Runout
0.5664684689	700000	Runout
0.8994913287	126202.26679	Failure
0.9456118649	91803.16881	Failure
0.7734093221	182878.88157	Failure
0.6228939956	625599.43961	Failure
0.7541963395	158932.9526	Failure
0.6704535148	295770.87888	Failure
0.6701621511	570697.91062	Failure
0.5394823078	700000	Runout
0.7119896139	335732.90245	Failure
0.6337254494	334474.16692	Failure
0.9616539956	63123.625627	Failure
0.6051927551	666992.16774	Failure
0.9732098115	61889.519465	Failure
0.6037298407	480453.42637	Failure
0.5692592836	700000	Runout
0.9744036355	87915.620718	Failure
0.7767297106	217653.90526	Failure



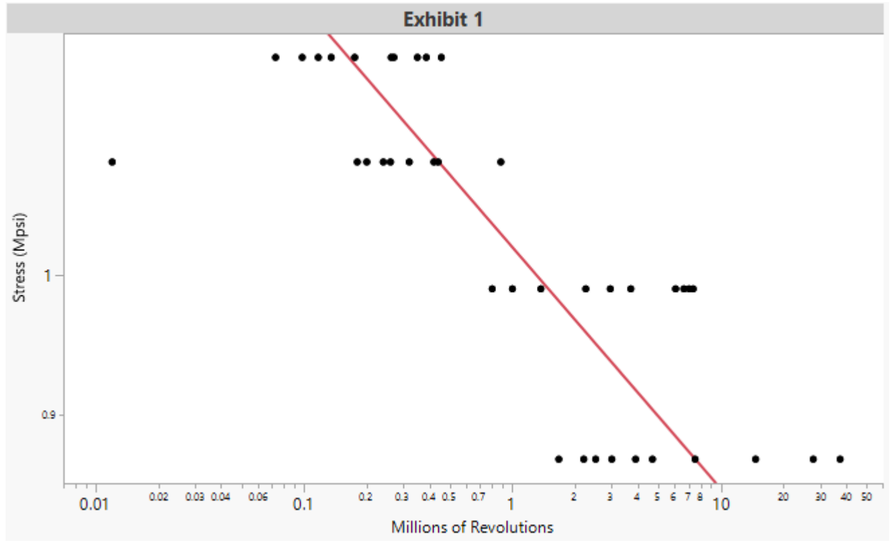
Stress is on Y. (log-scale)

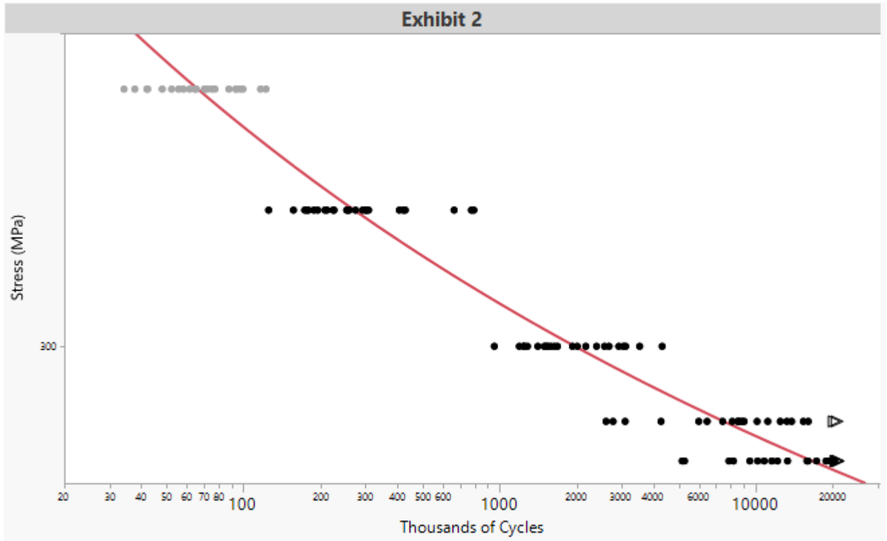
Cycles is on X. (log-scale)

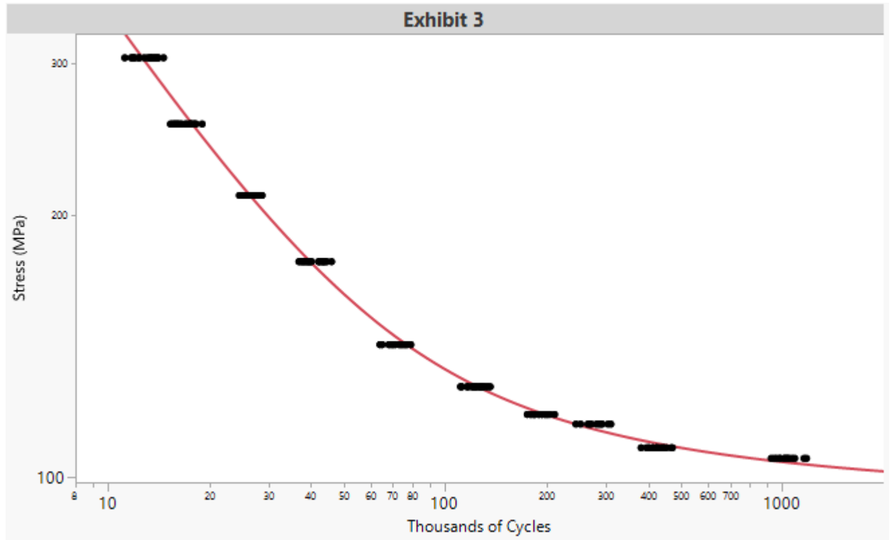
- Dots are failures.
- ▷ Triangles are runouts.

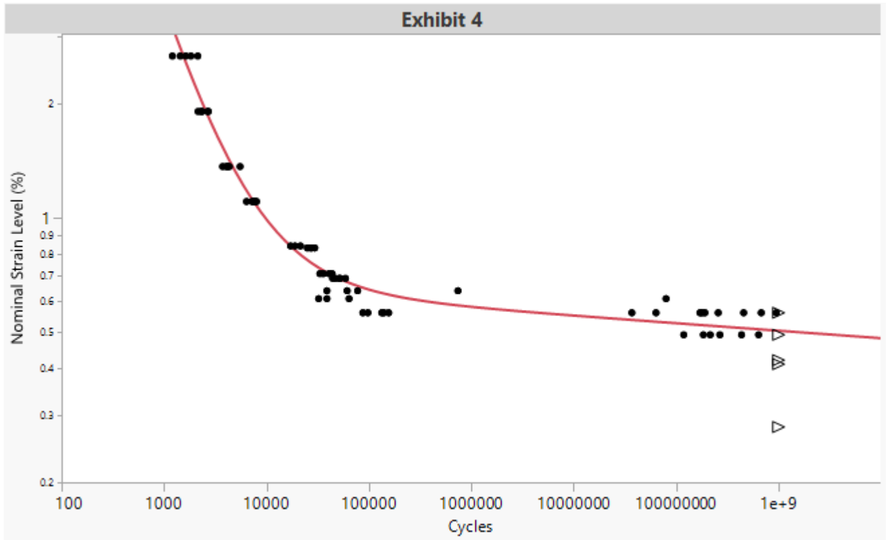
Data Characteristics I

Decreasing, may be curved









Data Characteristics II

Asymptotes

Exhibit 1

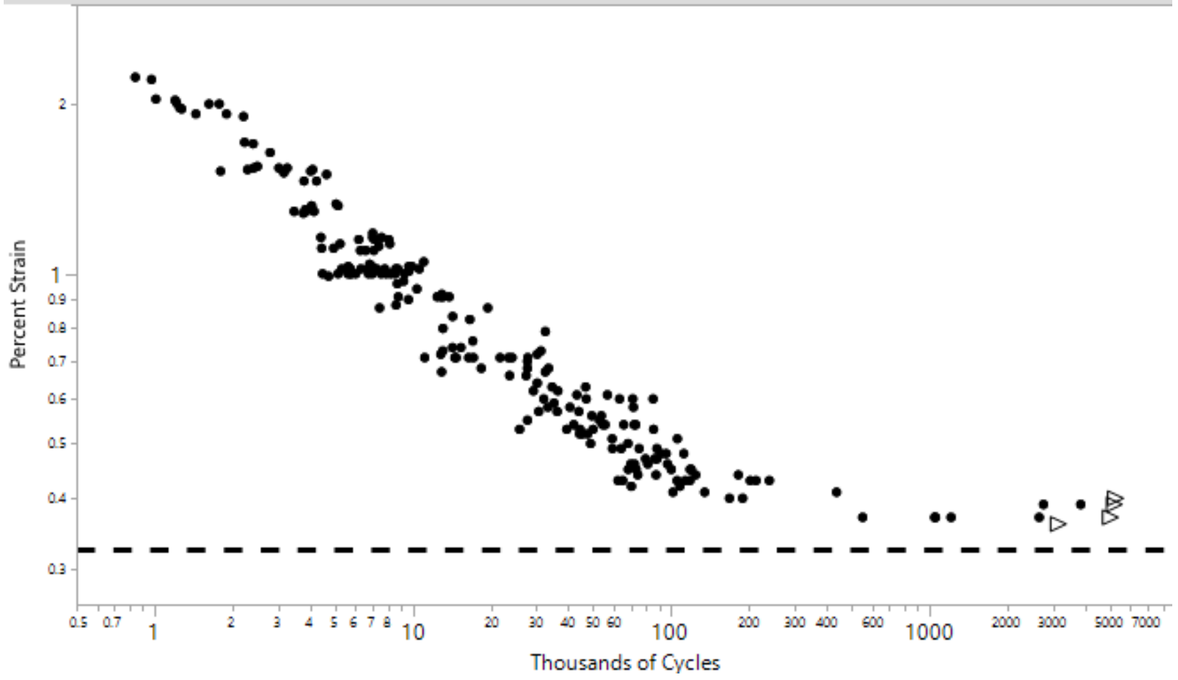
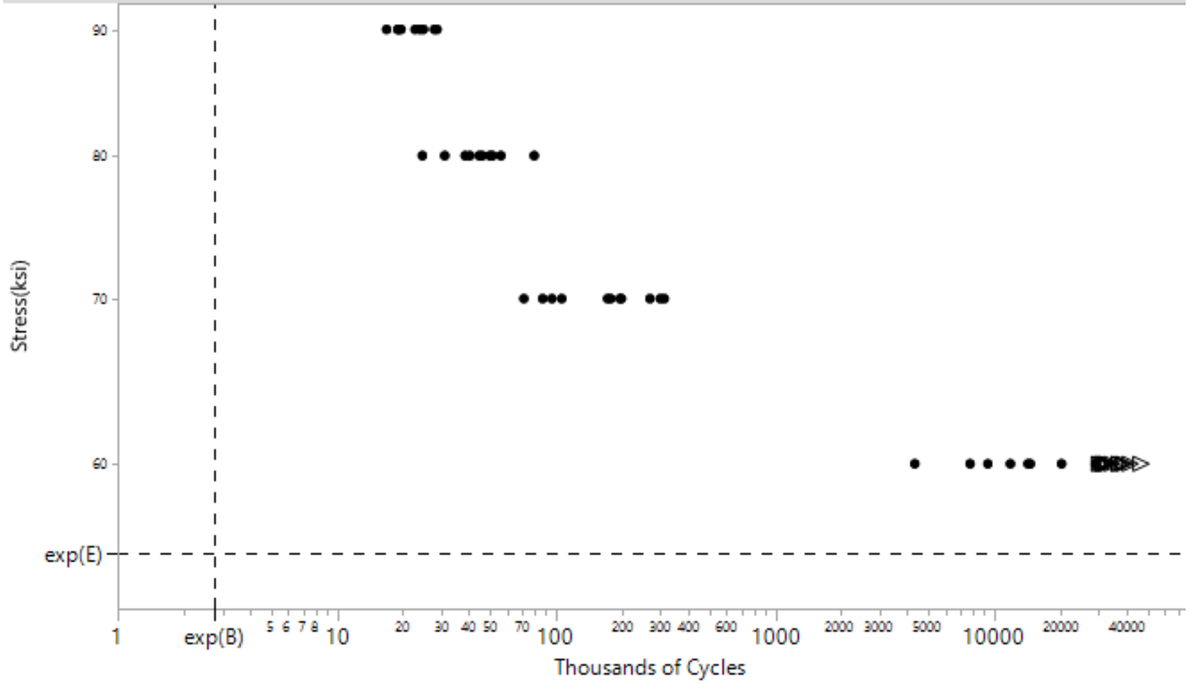
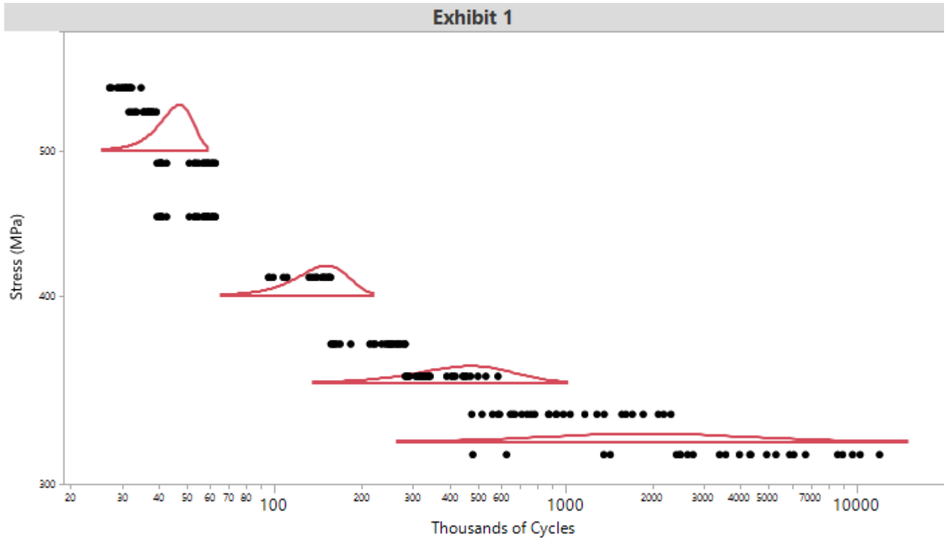


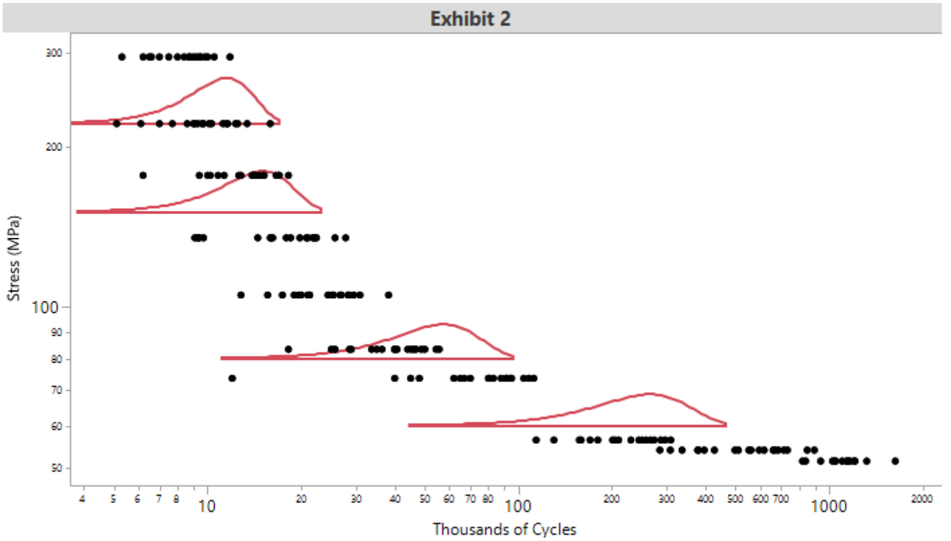
Exhibit 2

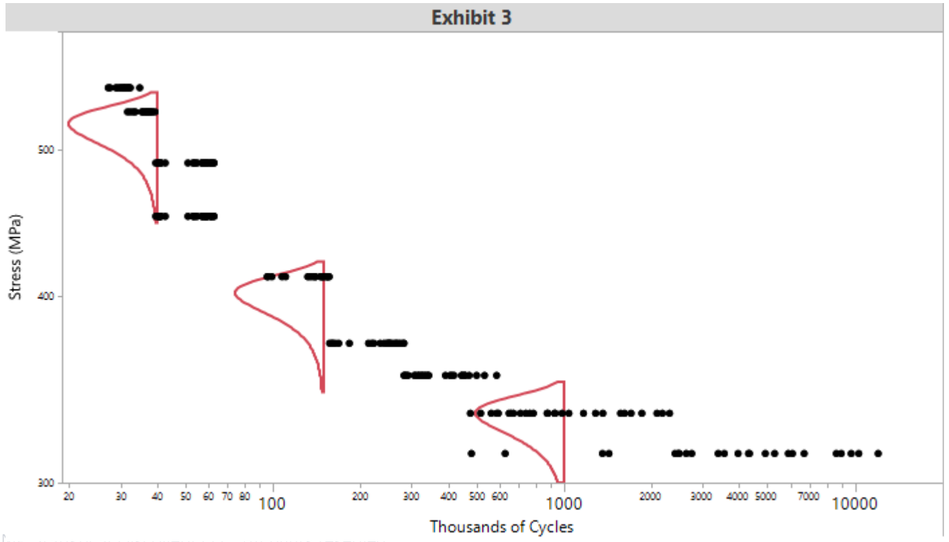


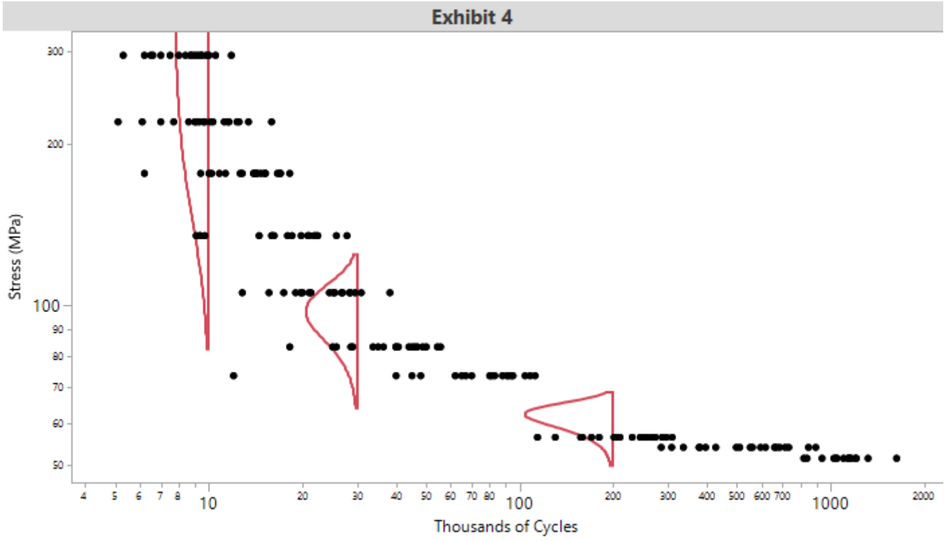
Data Characteristics III

Spread



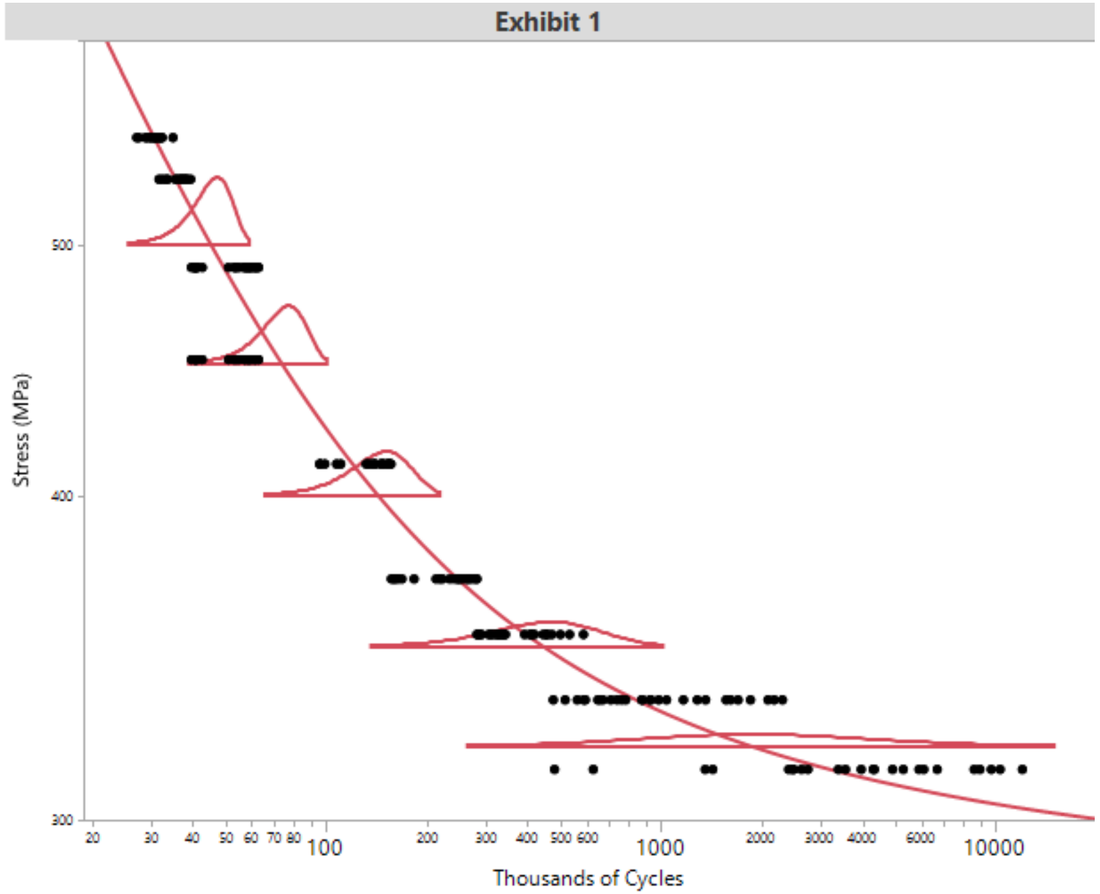


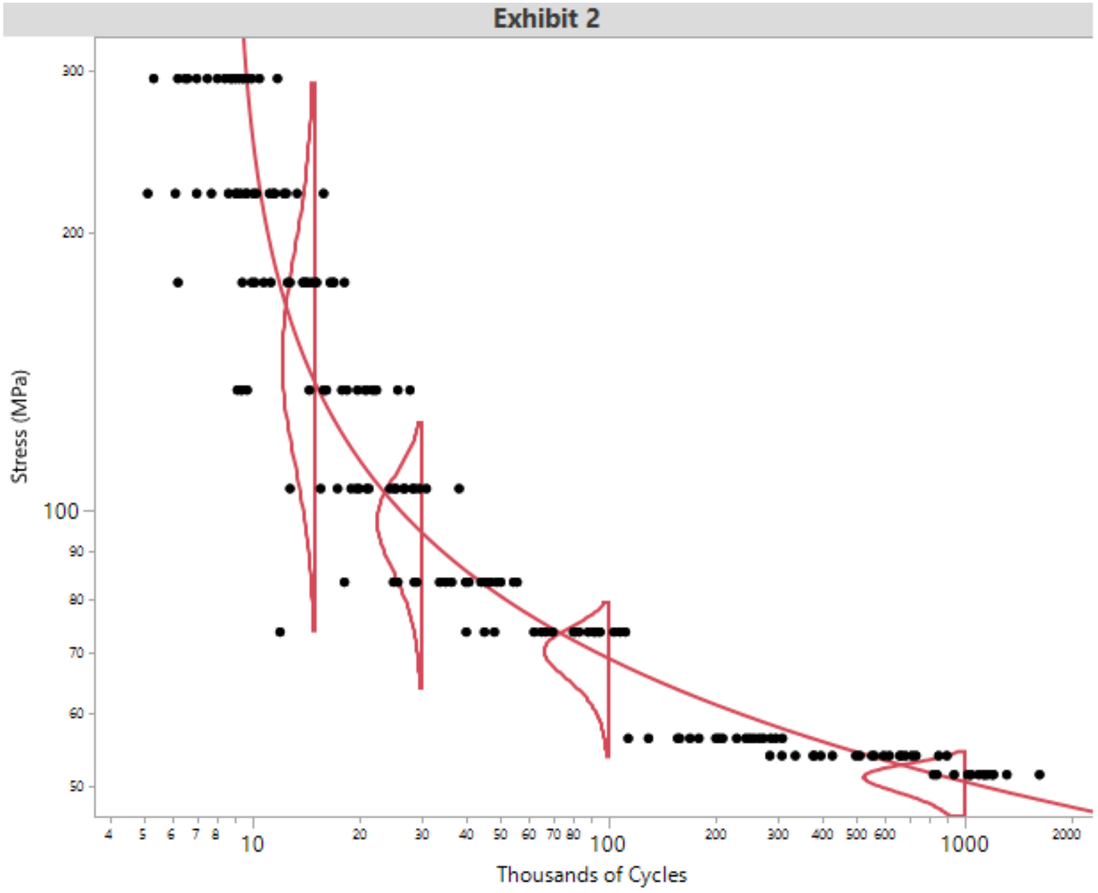




Objectives

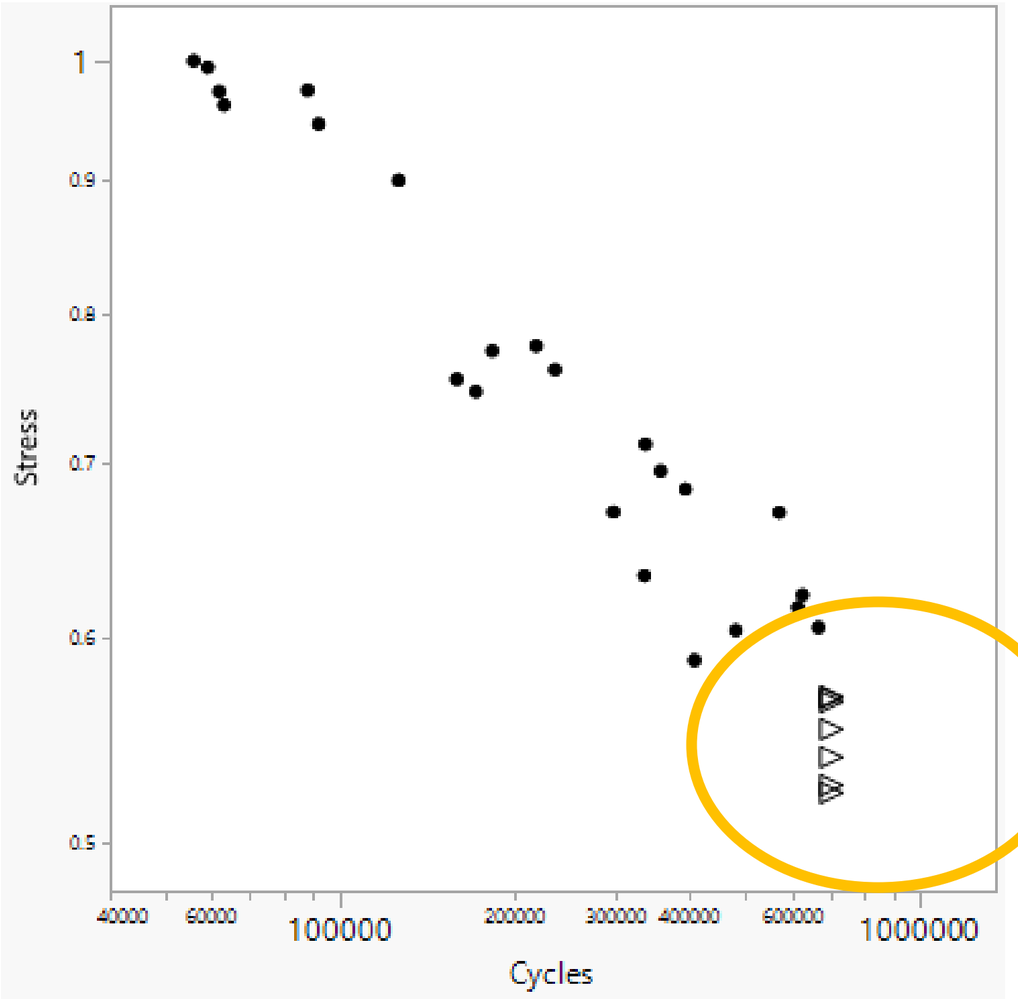
Fatigue-Life Distribution and Fatigue-Strength Distribution





Archaic Approach(s)

- Treat stress as the response, cycle as the explanatory, to estimate fatigue-strength distribution.
- Ignore runouts or treat them as failures.
- Use least squares with the presence of runouts.
- Model fatigue-life and fatigue-strength distributions separately.



Do it right (modern approach)

- ~~Treat stress as the response, cycle as the explanatory, to estimate fatigue-strength distribution.~~
- ~~Ignore runouts or treat them as failures.~~
- ~~Use least squares with the presence of runouts.~~
- ~~Model fatigue-life and fatigue-strength distributions separately.~~

- Cycle is the response; stress is the explanatory.
- Runouts are censored observations.
- Use Maximum Likelihood Inference all the time.
- Model fatigue-life and fatigue-strength distributions simultaneously, because they mutually determine each other.

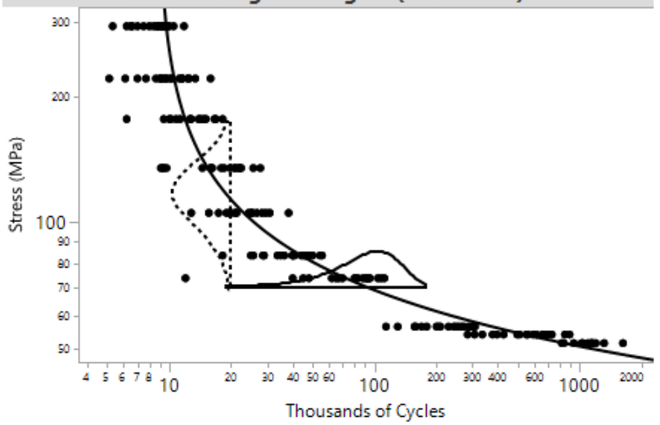
Models in JMP 18 Fatigue Model Platform

- Two categories, ***six*** S-N curve relationships :
  - Specify fatigue-life distribution, induce fatigue-strength distribution
    1. Box-Cox Loglinear-Sigma
    2. Basquin
  - Specify fatigue-strength distribution, induce fatigue-life distribution
    3. Coffin-Manson
    4. Nishijima
    5. Coffin-Manson Zero Elastic Slope
    6. Rectangular Hyperbola
- *Four* distribution types:
  - Lognormal, Weibull, Loglogistic, Frechet

24 models in total

Models in JMP 18 Fatigue Model Platform

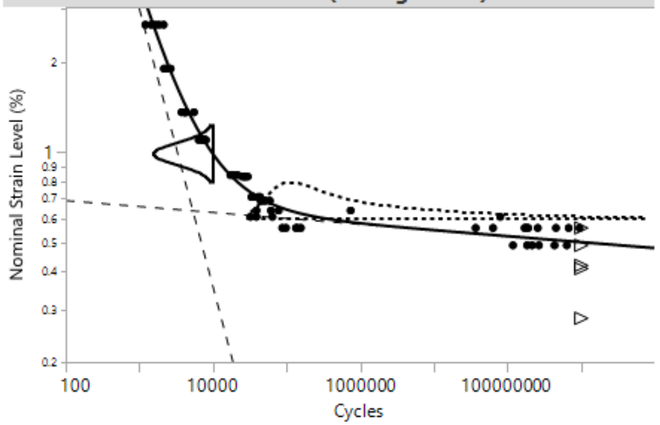
Box-Cox Loglinear Sigma (FL-Weibull)



Box-Cox Loglinear Sigma Formula

$$\mu = b0 + b1 \cdot \mathrm{BoxCox}\left(\mathrm{Stress}, \hat{\lambda}\right)$$

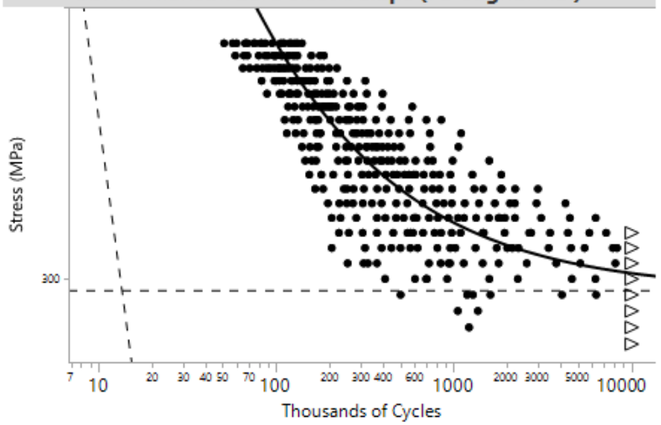
Coffin-Manson (FS-Lognormal)



Coffin-Manson Formula

$$\mathrm{Stress} = \mathrm{Ael} \cdot (2 \cdot N)^{\mathrm{bel}} + \mathrm{Apl} \cdot (2 \cdot N)^{\mathrm{cpl}}$$

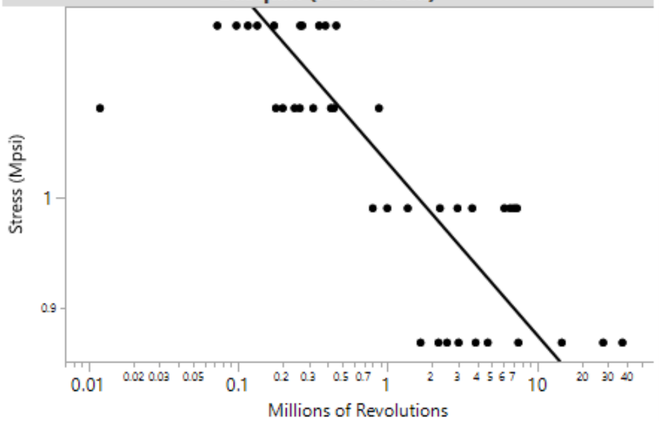
Coffin-Manson Zero Elastic Slope (FS-Lognormal)



Coffin-Manson Zero Elastic Slope Formula

$$\mathrm{Stress} = \mathrm{Ael} + \mathrm{Apl} \cdot (2 \cdot N)^{\mathrm{cpl}}$$

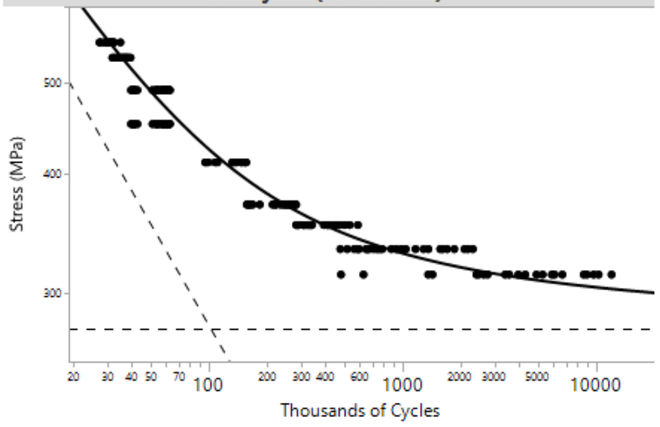
Basquin (FL-Weibull)



Basquin Formula

$$N = A \cdot \mathrm{Stress}^{(-B)}$$

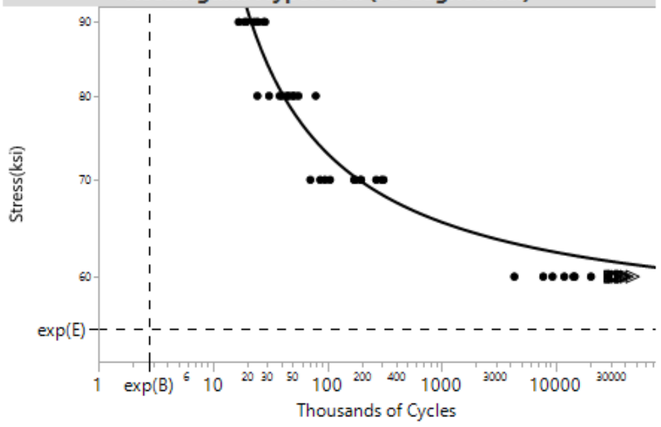
Nishijima (FS-Weibull)



Nishijima Formula

$$\mathrm{Stress} = \mathrm{Exp}\left(\frac{\left(-A \cdot \mathrm{Log}(N) + B + E + \sqrt{\left(A \cdot \mathrm{Log}(N) - (B - E)\right)^2 + 4 \cdot C}\right)}{2}\right)$$

Rectangular Hyperbola (FS-Lognormal)



Rectangular Hyperbola Formula

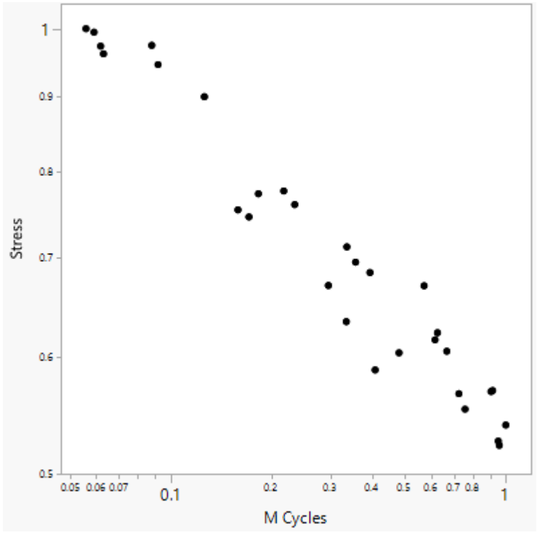
$$\mathrm{Stress} = \mathrm{Exp}\left(\frac{C}{(\mathrm{Log}(N) - B)} + E\right)$$

Available results from fitted models

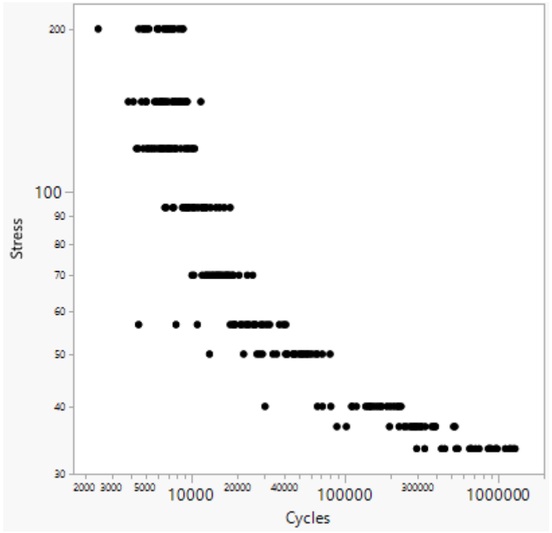
- Plot of data
- Model summary (parameter estimates etc.)
- Plot of density curves, quantile curves for fatigue-life/strength distributions
- Fatigue-life/strength cdf and quantile profilers
- Custom estimation for fatigue-life/strength cdf and quantile with likelihood-based confidence intervals

Demo

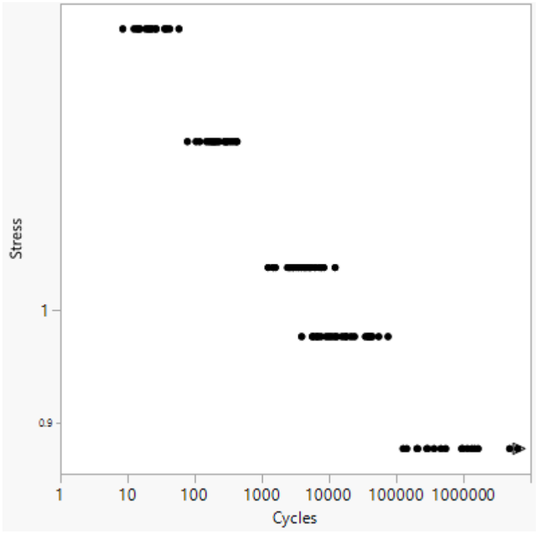
Sample data in JMP (under Help > Sample Index, search file name.)



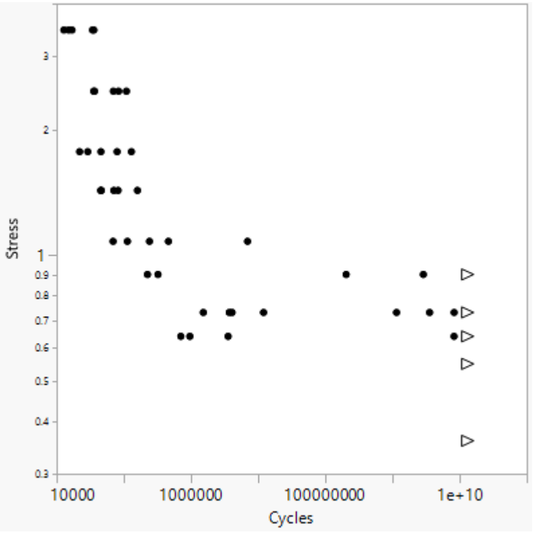
Metal Cable W.jmp
Basquin



Metal Wire X.jmp
Box-Cox



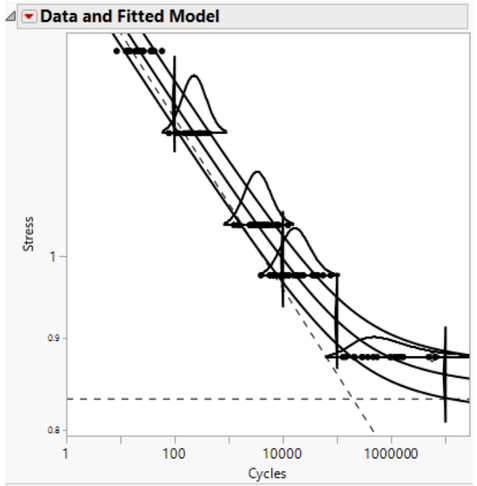
Metal Beam Y.jmp
Nishijima

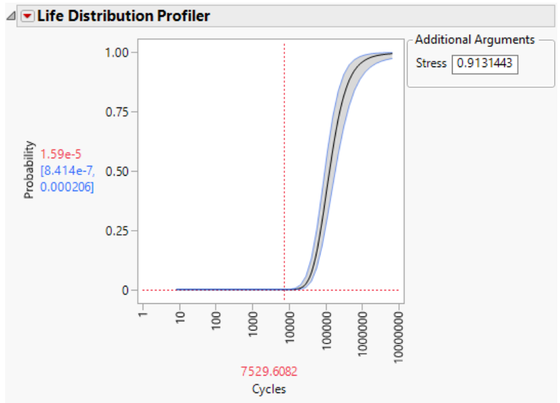


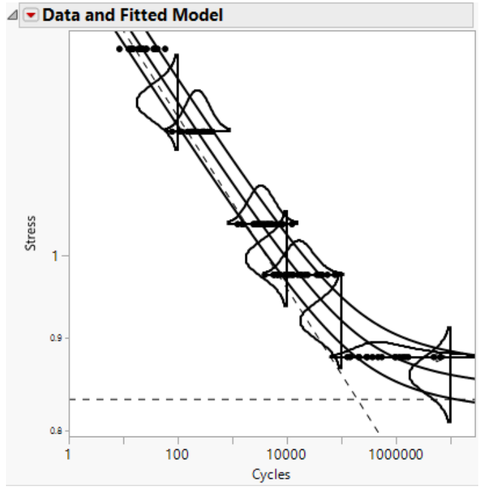
Metal Wire Z.jmp
Coffin-Manson

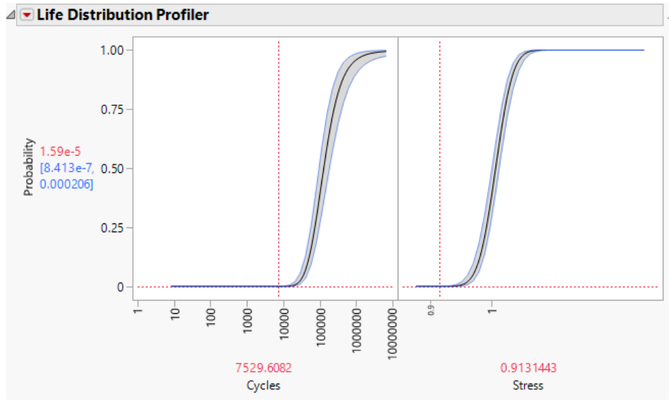
Slight updates in JMP 18.1

- The density curves are better proportioned.
- Distribution and quantile profilers use two frames instead of frame + num.









Thank you!

Peng Liu, Principal Research Statistician Developer
Rajneesh Rajneesh, Sr Analytics Software Tester