Design of Experiments Example: A Mixture of Mixtures Design

In this example, construct a design for a mixture of mixtures situation.

Consider the ingredients that go into a cake. Dry ingredients include flour, sugar, and cocoa. Wet ingredients include milk, melted butter, and eggs. The wet and dry components of the cake are two mixtures that are first mixed separately and then blended together. Table 7.1 lists the factors and the ranges over which you vary them as part of your experiment.

Table 7.1 Dry and Wet Components and Experimental Ranges

Mixture	Ingredient	Lower Level	Upper Level
Dry	Cocoa	10% (0.1)	20% (0.2)
	Sugar	0% (0)	15% (0.15)
	Flour	20% (0.2)	30% (0.3)
Wet	Butter	10% (0.1)	20% (0.2)
	Milk	25% (0.25)	35% (0.35)
	Eggs	5% (0.05)	20% (0.20)

The dry components (the mixture of flour, sugar, and cocoa) comprise 45% of the combined mixture. The wet components (butter, milk, and eggs) comprise 55%.

The goal of your experiment is to optimize a Taste rating. Taste is rated on a scale of 1 to 10, with 10 representing the best taste.

You construct a 10-run design to fit a main effects model. Because of the constraint on the proportions of dry and wet ingredients, you need to include only five factors in the Model outline to avoid singularity. The choice of which factor not to include is arbitrary.

Create the Design

1. Select **DOE > Custom Design**.
2. Double-click Y under Response Name and type **Taste**.

   Note that the default goal is Maximize. Because you want to maximize the **Taste** rating, do not change the goal.
3. Click under Lower Limit and type 0.

   The least desirable rating is 0.
4. Click under Upper Limit and type 10.

   The most desirable rating is 10.

5. Leave the area under Importance blank.

    Because there is only one response, that response is given Importance 1 by default.

6. From the Custom Design red triangle menu, select **Load Factors**.

7. Open Cake Factors.jmp from the Design Experiment sample data folder.

Figure 1 Completed Responses and Factors Outlines

Note that the factors are all mixture factors. The Values that define the range of settings for the experiment vary from factor to factor.

8. In the Define Factor Constraints outline, select **Specify Linear Constraints**.

9. In the Linear Constraints panel, click **Add** twice.

10. Enter the constraints shown in Figure 2.

    For the second constraint setting, be sure to select the greater than or equal to direction.

Figure 2 Define Factor Constraints

The two inequalities ensure that the dry factors sum to exactly 45% of the total, ensuring that the wet factors constitute the remaining 55%.

11. In the Model outline, select any effect and click **Remove Term**.

    Because of the equality constraint, a model containing all six effects would be singular.

12. Type 10 next to **User Specified**.

    Your experiment requires baking 10 cakes.

    Note: Setting the Random Seed in step 13 and Number of Starts in step 14 reproduces the exact results shown in this example. In constructing a design on your own, these steps are not necessary.

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13. (Optional) From the Custom Design red triangle menu, select **Set Random Seed**, type 1992991263, and click **OK**.

14. (Optional) From the Custom Design red triangle menu, select **Number of Starts**, type 40, and click **OK**.

15. Click **Make Design**.

    A JMP Alert informs you that your factor constraints include an equality constraint. This was what you intended, because the sum of the dry ingredient proportions is constrained to 45%.

16. Click **OK** to dismiss the JMP Alert.

17. Click **Make Table**.

Figure 3 Mixture of Mixtures Design

	Cocoa	Sugar	Flour	Butter	Milk	Eggs	Taste
1	0.1	0.15	0.2	0.15	0.35	0.05	•
2	0.1	0.05	0.3	0.1	0.25	0.2	•
3	0.1	0.15	0.2	0.1	0.25	0.2	•
4	0.2	0.05	0.2	0.1	0.25	0.2	•
5	0.2	0.05	0.2	0.1	0.35	0.1	•
6	0.1	0.05	0.3	0.2	0.3	0.05	•
7	0.15	0	0.3	0.1	0.35	0.1	•
8	0.2	0.05	0.2	0.2	0.25	0.1	•
9	0.1	0.15	0.2	0.2	0.25	0.1	•
10	0.2	0	0.25	0.2	0.25	0.1	•

Each run sums to 0.45.   Each run sums to 0.55.

The settings for the dry ingredients sum to 45% of the mixture and the settings for the wet ingredients sum to 55% of the mixture. The settings also conform to the upper and lower limits given in the Factors outline.

Analyze the Experimental Results

The Cake Data.jmp sample data table shows the results of the experiment. (Note that the runs in Cake Data.jmp are the same as those shown in Figure 3, but in a different order.) The design

table contains a Model script that opens a Fit Model window showing the five main effects specified in the DOE window's Model outline. Notice that the main effect of Egg is not included in the Model outline for this design. This script was saved to the data table when it was created by Custom Design.

1. Open the Cake Data.jmp sample data table, located in the Design Experiment folder.

2. In the Tables panel of the design table, click the red triangle next to **Model** and select **Run Script**.

    The main effect due to Egg is not included because it was excluded from the Model outline in the Custom Design window. All five effects are designated as Response Surface and Mixture effects.

3. Click **Run**.

    A JMP Alert appears, notifying you that the Profiler cannot be shown because of the additional constraint.

4. Click **OK** to dismiss the JMP Alert.

The Parameter Estimates report indicates that Sugar, Flour, and Butter are significant at the 0.05 level.

Figure 4 Parameter Estimates Report

Term	Estimate	Std Error	t Ratio	Prob>|t|
(Cocoa-0.1)/0.35	7.4556695	3.264525	2.28	0.0712
Sugar/0.35	14.141998	2.69585	5.25	0.0033*
(Flour-0.2)/0.35	12.957954	3.603964	3.60	0.0156*
(Butter-0.1)/0.35	11.457133	4.038675	2.84	0.0364*
(Milk-0.25)/0.35	6.0783774	3.853774	1.58	0.1756