Design of Experiments Example: A Two-Way Split-Plot Experiment

A two-way split-plot (also known as strip-plot or split-block) design consists of two split-plot components. In industry, these designs arise when batches of material or experimental units from one processing stage pass to a second processing stage. To use a two-way split-plot design, you must be able to reorder the units between stages.

After the first processing stage, you must be able to divide the batches into sub-batches. The second-stage processing factors are applied randomly to these sub-batches. For a specific second-stage experimental setting, all of the sub-batches assigned to that setting can be processed simultaneously. Additional factors can be applied to experimental units after the second processing stage.

In contrast to a split-split-plot design, the second-stage factors are *not nested* within the first-stage factors. After the first stage, the batches are subdivided and formed into new batches. Therefore, both the first- and second-stage factors are applied to whole batches.

Although factors at both stages might be equally hard-to-change, in order to distinguish these factors, JMP denotes the first stage factors as *very-hard*-to-change and the second-stage factors as *hard*-to-change. Additional factors applied to experimental units after the second processing stage are considered *easy*-to-change.

Scenario for a Two-Way Split-Plot Design

This example is based on an experiment to improve the open circuit voltage (OCV) in battery cells (Vivacqua and Bisgaard, 2004). You need to minimize the OCV in order to keep the cells from discharging on their own.

Battery cells move through two stages of processing:

- First stage: A continuous assembly process where batteries are processed in batches of 2000.
- Second stage: A curing process with a 5-day cycle time in a chamber that can accommodate 4000 batteries.

You want to study six two-level continuous factors:

- Four factors (A1, A2, A3, and A4) are applied to the assembly process. You can run 16 trials for the first-stage factors.
- Two factors (C5 and C6) are applied to the curing process. Because curing requires a 5-day cycle time, you can run only 6 cycles (30 days) for the second-stage factors. Using six curing cycles gives you partial replication of the curing settings, enabling you to test for curing effects.

Both the first- and second-stage factors are hard-to-change, suggesting two split-plots. However, the batches of 2,000 batteries from the first-stage experiment can be divided into sub-batches of 500 batteries each. Eight of these sub-batches can be randomly selected and processed simultaneously in the curing chamber.

The experiment has 48 experimental units. Note that the first- and second-stage factors are *crossed*.

Create the Design

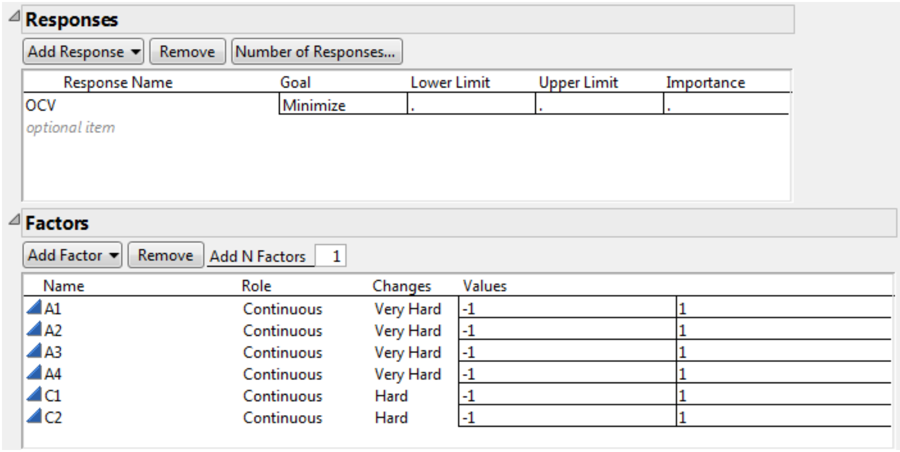
To design a two-way split-plot experiment:

1. Select **DOE > Custom Design**.
2. Double-click Y under Response Name and type OCV.
3. Under Goal, click **Maximize** and select **Minimize**.
4. To add factors manually, follow step 5 through step 10. Or, to load factors from a saved table:
   a. Select **Load Factors** from the Custom Design red triangle menu.
   b. Open the Battery Factors.jmp sample data table, located in the Design Experiment folder.
   c. Proceed to step 11.
5. Type 6 next to **Add N Factors**.
6. Click **Add Factor > Continuous**.
7. Rename the factors A1, A2, A3, A4, C1, and C2.

   Keep the default Values of -1 and 1 for these factors.
8. For each of the factors A1, A2, A3, and A4, under Changes, click **Easy** and change it to **Very Hard**.

   To distinguish between the first- and second-stage factors, you designate the Changes for the first-stage factors as Very Hard, and the Changes for the second-stage factors as Hard.
9. For each of the factors C1 and C2, under Changes, click **Easy** and change it to **Hard**.

Figure 1 Responses and Factors Outlines



10. Click **Continue**.
11. Select **Interactions> 2nd** in the Model outline.
12. In the Design Generation outline, select the option **Hard to change factors can vary independently of Very Hard to change factors**.

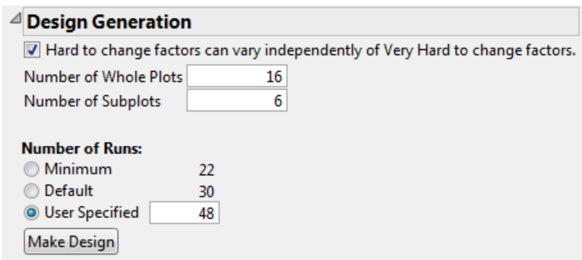
    See Figure 2. Checking this option creates a two-way split-plot design. If this option is not checked, the design is treated as a split-split-plot design, with nesting of factors at the two levels.
13. Type 16 as the **Number of Whole Plots**.

    This is the number of trials that you can run for the first-stage factors.
14. Type 6 as the **Number of Subplots**.

    This is the number of trials that you can run for the second-stage factors.
15. Under Number of Runs, type 48 next to **User Specified**.

    This is the total number of experimental units.

Figure 2 Design Generation Outline



Note: Setting the Random Seed in step 16 and Number of Starts in step 17 reproduces the exact results shown in this example. In constructing a design on your own, these steps are not necessary.

16. (Optional) From the Custom Design red triangle menu, select **Set Random Seed**, type 1866762673, and click **OK**.
17. (Optional) From the Custom Design red triangle menu, select **Number of Starts**, type 21, and click **OK**.
18. Click **Make Design**.
19. Click **Make Table**.

Figure 3 Partial View of Design Table

Custom Design
Design Custom Design
Criterion D Optimal
Screening
Model
DOE Dialog

Columns (9/0)
Whole Plots *
Subplots *
A1 *
A2 *
A3 *
A4 *
C1 *
C2 *
OCV *

Rows
All rows 48
Selected 0
Excluded 0
Hidden 0

	Whole Plots	Subplots	A1	A2	A3	A4	C1	C2	OCV
1	1	1	-1	1	-1	1	-1	1	•
2	1	2	-1	1	-1	1	1	1	•
3	1	3	-1	1	-1	1	1	-1	•
4	2	4	1	1	-1	1	1	1	•
5	2	5	1	1	-1	1	-1	1	•
6	2	6	1	1	-1	1	-1	-1	•
7	3	1	-1	-1	-1	1	-1	1	•
8	3	2	-1	-1	-1	1	1	1	•
9	3	3	-1	-1	-1	1	1	-1	•
10	4	4	-1	-1	-1	-1	1	1	•
11	4	5	-1	-1	-1	-1	-1	1	•
12	4	6	-1	-1	-1	-1	-1	-1	•
13	5	1	1	1	-1	-1	-1	1	•
14	5	2	1	1	-1	-1	1	1	•
15	5	3	1	1	-1	-1	1	-1	•
16	6	4	1	-1	-1	-1	1	1	•
17	6	5	1	-1	-1	-1	-1	1	•
18	6	6	1	-1	-1	-1	-1	-1	•
19	7	1	1	-1	1	-1	-1	1	•
20	7	2	1	-1	1	-1	1	1	•
21	7	3	1	-1	1	-1	1	-1	•
22	8	4	-1	1	-1	-1	1	1	•
23	8	5	-1	1	-1	-1	-1	1	•
24	8	6	-1	1	-1	-1	-1	-1	•

The design table shows 16 levels for Whole Plots. For each level of Whole Plots, the settings of the four assembly factors are constant. From each level of Whole Plots, three batches of 500 batteries (Subplots) are randomly assigned to settings of the curing factors. Two sets of curing conditions are replicated (C1 = -1, C2 = 1 and C1 = 1, C2 = 1). To see this, select columns C1 and C2, right-click in the header area, and select **Sort > Ascending**.

Analyze the Results

The Battery Data.jmp sample data table contains experimental results for the design that you generated.

1. Select **Help > Sample Data Library** and open Design Experiment/Battery Data.jmp.
2. From the Model red triangle, select **Run Script**.

   Notice the following in the Fit Model window:

   – The factor Whole Plots has the Attribute called Random Effects (**&Random**). This specifies that the levels of Whole Plots are random realizations. They have an associated error term.

   – The factor Subplots also has the Random Effects Attribute (**&Random**).

   – The analysis Method is **REML (Recommended)**. This method is specified precisely because the model contains random effects. For more information about REML models, see the *Fitting Linear Models* book.

Tip: In the Fit Model window, JMP Pro users can change the Personality to Mixed Model.

3. Check the option to **Keep dialog open**.
4. Click **Run**.

Figure 4 Report for Full Model

Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|
Intercept	38.183196	1.673404	2.075	22.82	0.0016*
A1	-4.657571	1.093074	2.916	-4.26	0.0251*
A2	0.147742	1.093074	2.916	0.14	0.9013
A3	0.9486795	1.093074	2.916	0.87	0.4510
A4	4.0716925	1.190175	3.214	3.42	0.0376*
C1	-1.827487	1.606207	0.868	-1.14	0.4804
C2	-15.73288	1.35446	0.719	-11.62	0.1065
A1*A2	0.1289012	1.080526	2.784	0.12	0.9131
A1*A3	-1.114015	1.080526	2.784	-1.03	0.3837
A1*A4	-1.92398	1.176342	3.04	-1.64	0.1993
A1*C1	-3.271232	0.480531	20.65	-6.81	<.0001*
A1*C2	3.737258	0.478635	20.09	7.81	<.0001*
A2*A3	0.3768179	1.080526	2.784	0.35	0.7520
A2*A4	1.4693535	1.176342	3.04	1.25	0.2992
A2*C1	1.9072313	0.480531	20.65	3.97	0.0007*
A2*C2	-0.161179	0.478635	20.09	-0.34	0.7398
A3*A4	0.0764369	1.176342	3.04	0.06	0.9522
A3*C1	0.1761619	0.480531	20.65	0.37	0.7176
A3*C2	-0.445242	0.478635	20.09	-0.93	0.3633
A4*C1	0.5306856	0.511896	20.84	1.04	0.3118
A4*C2	1.3506119	0.502894	20.12	2.69	0.0142*
C1*C2	-0.425326	1.606207	0.868	-0.26	0.8401

Random Effect Predictions

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Pct of Total
Whole Plots	1.6095523	15.451509	15.876576	-15.66601	46.569027	45.968
Subplots	0.8919389	8.562507	16.314188	-23.41271	40.537728	25.473
Residual		9.5998804	3.0299487	5.6239301	19.985723	28.559
Total		33.613897	16.824615	15.325965	123.59174	100.000

-2 LogLikelihood = 227.810084

Note: Total is the sum of the positive variance components.

Total including negative estimates = 33.613897

The Parameter Estimates report indicates that four two-way interactions, A1*C1, A1*C2, A2*C1, and A4*C2, and two main effects, A1 and A4, are significant at the 0.05 level.

5. In the Battery Data.jmp table, from the red triangle next to **Reduced Model 1**, select **Run Script**.

   The script opens a Fit Model window where insignificant interactions have been removed. The remaining effects are all main effects and the four two-way interactions A1*C1, A1*C2, A2*C1, and A4*C2. You are reducing the model in a conservative fashion.

6. Click **Run**.

Figure 5 Report for Preliminary Reduced Model

Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|
Intercept	38.249531	1.373127	7.723	27.86	<.0001*
A1	-4.723906	1.175023	8.5	-4.02	0.0034*
A2	0.0497917	1.165418	8.228	0.04	0.9669
A3	0.7560417	1.165418	8.228	0.65	0.5342
A4	4.2328205	1.207654	9.16	3.50	0.0065*
C1	-2.195626	0.866042	1.083	-2.54	0.2234
C2	-15.79922	0.840903	1.008	-18.79	0.0331*
A1*C1	-3.283225	0.452505	25.17	-7.26	<.0001*
A1*C2	3.8035938	0.4498	24.42	8.46	<.0001*
A2*C1	1.8797279	0.452505	25.17	4.15	0.0003*
A4*C2	1.3877006	0.482556	24.95	2.88	0.0081*

Notice that the main effect C2 is now significant at the 0.05 level (Prob>|t| = 0.0331)

7. In the Fit Model window, remove A3.

   The main effect A3 is the only main effect that is not significant and not involved in a two-way interaction.

8. Click **Run**.

Figure 6 Report for Reduced Model

Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|
Intercept	38.249531	1.362184	6.639	28.08	<.0001*
A1	-4.723906	1.137313	9.131	-4.15	0.0024*
A2	0.0497917	1.127389	8.818	0.04	0.9658
A4	4.2131272	1.173862	9.921	3.59	0.0050*
C1	-2.166553	0.899138	1.037	-2.41	0.2432
C2	-15.79922	0.874257	0.963	-18.07	0.0389*
A1*C1	-3.282201	0.452234	25.24	-7.26	<.0001*
A1*C2	3.8035938	0.449735	24.43	8.46	<.0001*
A2*C1	1.8843473	0.452234	25.24	4.17	0.0003*
A4*C2	1.3851686	0.4823	24.91	2.87	0.0082*

Random Effect Predictions

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Pct of Total
Whole Plots	2.0231563	17.459497	9.7154555	-1.582446	36.50144	60.025
Subplots	0.3473621	2.9976761	5.8732929	-8.513766	14.509119	10.306
Residual		8.6298312	2.4692519	5.2816799	16.588508	29.669
Total		29.087005	9.1941655	17.028461	60.633509	100.000

-2 LogLikelihood = 254.17861349

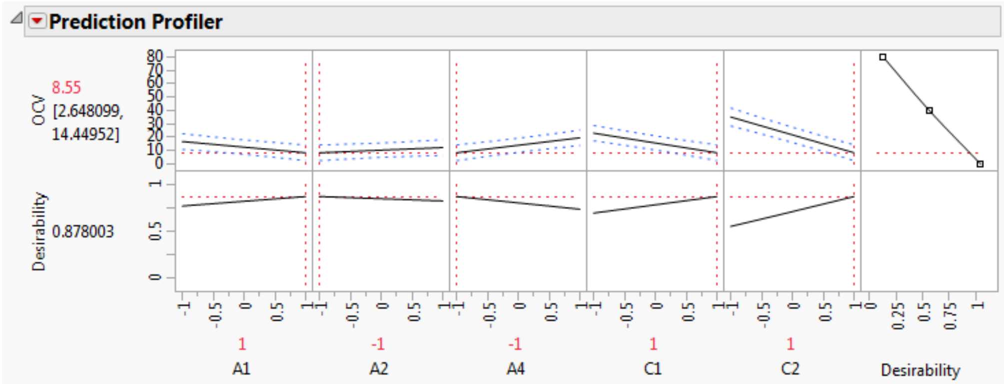
Note: Total is the sum of the positive variance components.
Total including negative estimates = 29.087005

The REML Variance Component Estimates report shows that the variance component associated with Whole Plots is about six times as large as the variance component for Subplots. This suggests that the assembly process is more variable than the curing process. Also, the within (Residual) error is larger than that for Subplots. Efforts to reduce variation should focus on the assembly process and on battery-to-battery differences.

9. From the red triangle next to Response OCV, select **Factor Profiling > Profiler**.

10. From the red triangle next to Prediction Profiler, select **Maximize Desirability**.

Figure 7 Prediction Profiler with Settings That Minimize OCV



The profiler shows the five factors identified as active and settings that minimize OCV.